OFFSHORE STRUCTURAL SYSTEMS RELIABILITY

Prepared by

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EXECUTIVE SUMMARY

The objective of this report on structural systems reliability is to compare the offshore industry's current needs with the available computational methods. The major part of the report, however, lays the ground work by introducing the concepts and implications of structural systems reliability to the offshore engineer who is versed in structural behavior and elementary reliability notions. Systems reliability promises to integrate design, inspection, operations, and economics to enhance the productivity and safety of offshore facilities. The rapid development and spread of probabilistic representations of loads, materials, and member behavior in structures (as evidenced in existing and anticipated probability-based industry codes) makes it urgent that these concepts and their benefits be extended beyond the individual member and into the structural system as a whole.

We are interested in the probabilistic behavior of the total structure, including the relationship between the reliability of individual members, their post-failure mechanical behavior, and the reliability of the structural system as a whole. Structural systems reliability focuses upon issues such as redundancy, robustness with respect to damaged members, and the role of inspection.

In Chapter 2 an explicit formulation is presented for the failure probability of the system, starting from familiar elements such as the annual probability of exceeding the current design load through the first-failure of any component in a large system to factors associated with the system's static redundancy and residual strength. The formulation shows these system effects in the context of both overloads and exogenously caused accidental damages (such as fabrication errors and ship collisions). The explicit role of inspection (both quality and interval) is represented in this formulation. Several new factors, called complexity, redundancy and robustness, are introduced to isolate and emphasize the particular characteristics of a structural system as opposed to an individual member.

The third chapter is designed as a primarily qualitative, but fundamental introduction to the elementary concepts of the reliability of determinate and indeterminate structural systems. The system failure probability is related to the member-failure events, to the random safety margins of elements, and to the individual members' random capacities and load effects. Without any calculations the reader should begin to understand well the impacts on structural system safety of such notions as the degree of static indeterminacy, the post-failure behavior of members (i.e., brittleness versus ductility), the role of well-balanced versus unbalanced structural designs, and finally, more subtle issues such as the degree of stochastic correlation among the failure events. The deduced conclusions are also backed up by some numerical results for concreteness.

The report turns next to explicit examples including, in particular, the results of a case study conducted by the project with the cooperation of several of the participants, in particular, Statoil, Exxon Production Research, PMB Systems, and Chevron. Through the application of systems reliability analysis to an eight-leg steel jacket platform under wave loadings we are able to demonstrate quantitatively notions such as complexity, redundancy, and robustness in a realistic structure. By making changes to the system and repeating the analysis, we are able to begin to quantify the role of ductility, the impact of system-layout (K- versus X-bracing system), secondary-member sizing rules, location of the structure, and finally, robustness with respect to loss or damage of members.

The industry needs for systems reliability are categorized into those for new, novel platform types, for design of current platform types, and for operating, existing platforms. Novel structures can benefit through increased understanding of the hazards they face, their comparative robustness/redundancy, and the degree and impact of the uncertainties implied by novelty. Design of current platform types can be improved by systems reliability evaluations of alternate framing concepts and member-sizing procedures. Operating platforms may benefit from more rational, systems-based inspection procedures, damage impact evaluations, and reassessments. Examples include proposed deck load upgrades and development of inspection and repair strategies for early generations of platforms.

Currently, analysis methods are available for efficient estimation of the reliability of typical platforms under static push-over loadings. The mechanical models are rather restrictive, however. Other special structural problems have been studied in greater or lesser detail but the analysis methods must be fashioned each time to the problem at hand, requiring a systems reliability analyst's assistance. Little experience yet exists with respect to more complex loadings such as static or dynamic load reversals. Also the role of inspection and fatigue damage need to be studied in the broader context of platform systems reliability.

Comparing these needs and current capabilities, one observes a gap between the broad, expanding use in the offshore industry of probabilistic representation of loads, materials, and design criteria, on one hand, and the non-trivial, but still limited ability to consider full structural systems-level interactions and implications on the other. The implications are that we should seek to enhance the mechanical modeling assumptions underlying the current efficient methods, develop more general-purpose reliability analysis capabilities, and promote the continuation and expansion of special studies of systems-level issues by many individuals in the industry.

OFFSHORE STRUCTURAL SYSTEMS RELIABILITY

Chapter 1

STRUCTURAL RELIABILITY IN THE OFFSHORE INDUSTRY

1.1 Introduction

Structural systems reliability promises to be the tool that will bring together the several key elements in offshore structures safety, operability, and economy. The recent NRC "DIRT" conference (*U.S. National Research Council, 1984*) referred to these elements as the "Design, Inspection, and Redundancy" Triangle. The conference called for expansion of the new reliability analysis techniques out of their current emphasis on member reliability and into the realm of system reliability. Baker (*1985*) states that systems reliability analysis promises to impact design data analysis, structural analysis, specifications, quality control, inspection, defect assessment, repair, monitoring, and maintenance in the industry. This report will explore the industry's needs with respect to structural systems reliability and the available methods with the objective of drawing conclusions as to the necessary developments in systems reliability for its prompt and useful application in the offshore industry.

Structural systems reliability (SSR) represents the intersection among the three fields of structural reliability, structural systems, and systems reliability. By *structural reliability* we mean simply the field of probabilistic analysis of structural behavior, serviceability, and safety. By *structural systems analysis* we mean to emphasize the behavior of the structure as a whole as opposed to the behavior of individual cross-sections, components, members or elements of that system. For example, in structural systems analysis one becomes concerned with post-yielding or post-failure behavior of structural elements because this determines the impact of a member's failure upon the system's behavior. As a second example, structural systems implies the concern with system-level ductility as opposed to element-level ductility; through redundancy one can obtain the former even without the latter. *Systems reliability* (without the specification structural) is that field of probabilistic reliability theory that deals with the relationship between system behavior and component behavior. The so-called PRA (probabilistic risk assessment) in the nuclear power plant engineering field is an excellent example of the recent successful engineering application of systems reliability.

Structural systems reliability then is the study of the probabilistic behavior of total structures. In this topic one is interested in the relationship between the states (e.g., failed or not failed) of members and the state of the system. More fully, one is interested in how the uncertainties about the states of the elements are transformed into uncertainty about the state of the system. Typical questions answered by structural systems reliability include those about systems redundancy, which in this context might be defined as the probability of systems failure given first component failure (Section 2). Other important questions include the impact of lack of ductility in the post-failure behavior of components and connections upon the reliability of the system as a whole. In this and the following chapter, we will look more closely at the questions and answers that systems reliability promises.

This report will focus on the narrower subject of structural systems reliability as distinct from what we might call *full-scope* systems reliability analysis. In the offshore industry the latter term has implied the study of the full spectrum of design, construction, operations, inspection and maintenance of offshore facilities, of which the structure is only a part (or component) in this broader context. Such studies have been recommended by the Norwegian Petroleum Directorate (NPD) at the conceptual stage of platform design to improve layout, to identify accidental loads, and to assign reliability goals to safety related functions and procedures. In some full-scope systems analyses of existing facilities structural systems analysis is merely a subset. In the nuclear industry there is ample practical precedent for this kind of full-scope analysis incorporating detailed structural systems reliability; the PRA's mentioned above normally include structural and mechanical behavior within their analysis. A prime example is that of seismically-induced nuclear accidents. Such full-scope studies can even be primarily structurally directed. The author is aware of at least two major studies in which full-scope systems reliability analyses are being conducted with the prime objective of developing "top-down" specifications of the reliability allocation for individual structural systems and components. The first iteration leads to a set of design criteria for the structural/mechanical engineers. Their design is in turn assessed along with the other contributing reliability elements in the total problem to ascertain if the reliability and/or cost-benefit goals have been met. If not, the opportunity is available for subsequent iterations of the entire analysis and reliability allocation. One can envision such schemes including the full spectrum of structural design, inspection and maintenance.

1.2 The Status of Structural Reliability

Elements of structural reliability have already made a major impact upon the design and assessment of offshore structures. Virtually all design events are now specified in terms of a mean return period such as 100 years. Structural reliability was used to calibrate the safety coefficients in the 1977 NPD and Det Norske Veritas (DNV) regulations. A probability-based load and resistance factor design (LRFD) guideline is currently being drafted by the API (*Moses, 1986*). In the development of such codes professional committees incorporate information about the probabilistic nature of structural loads and materials as well as uncertainties in predictions of structural component behavior. The product, through calibration with existing codes, is a partial factor or a load and resistance factor format for design of structural members. The procedure being followed by the API committee parallels that associated with the development of recent structural building codes, e.g., the 1986 AISC Steel Building Code and the 1982 American National Standards Institute Building Loads Code (see, for example, *Ellingwood, et al., 1982*).

Some more recent codes have permitted formal structural reliability analysis as an alternative to conventional factor-based design codes. Some new structures have been analyzed in special studies by reliability analysis to ascertain whether their members have safety levels comparable to those in existing codes.

Such developments have encouraged in both offshore literature and practice the reporting of all important variables in terms of their probability distributions or, at a minimum, in terms of their means and coefficients of variation (COV). Such concepts have clarified the reporting and interpretation of randomness and uncertainty. For example, it is understood that the randomness in the maximum annual wave height in the North Sea is substantially less than that in the Gulf of Mexico; this is reflected in a substantially lower coefficient of variation. On the other hand, given the wave height, the uncertainty due to various factors in estimating the base shear on a steel jacket platform may be as high as a COV of 25 percent. As the industry has passed on to other new and unusual environments such as the Arctic, these techniques in load definition have formed the basis for the collection and interpretation of data. For example, in order to calculate the probability distribution of the forces upon an offshore facility that might be struck by an ice floe, it is necessary to collect and process information related to the mean occurrence rate and the joint probability distribution of size and speed of such features in the local environment.

Early assessment of the problem in the Arctic made it very clear that a major source of uncertainty was that contributed by the estimation of forces upon the structure given the impact of an ice floe of specified size and speed. In contrast to the (pure) randomness in the occurrence and size of the features, this uncertainty in the impact forces represents primarily uncertainty caused by our incomplete professional information. This last example demonstrates the need in all such reliability analysis to distinguish between what we shall call here "randomness" and "uncertainty." We will make this distinction more precise in a section to follow. One of the important aspects of uncertainty is that it is subject to updating, i.e., to its reduction through the collection of additional information about the loading environment and/or the structural elements and their behavior. The analysis of this updating is at the heart of any probabilistic formulation and interpretation of the critical problems of inspection and structure re-evaluation. Formal methods are available for such updating and have been applied to individual structural elements (e.g., *Madsen et al, 1986; Madsen, 1987*).

Whereas such probability-based assessments can be used in many relative safety assessment situations, to be most beneficial it has been found appropriate to incorporate them into a larger cost-benefit-risk evaluation (e.g., *Bea et al, 1984*). The purpose of such an analysis is to permit rational economic trade-offs between the increased initial cost of a stronger structure versus the reduced expected future cost associated with potential failure. Inspection strategy must necessarily involve cost and risk trade-offs. Such cost-benefit studies are now quite common in the Offshore Engineering literature and have provided a clarification of the essential design compromises facing the structural engineer.

The basic methodological techniques for all of these problems, at least as applied to the individual structural member, are in a rather high state of theoretical development and are under accelerated usage in offshore practice. It is not the objective of this report to cover this entire spectrum of structural reliability theory. Rather our focus is on structural systems reliability, a topic which also encompasses all of these elements: loading, material and member behavior, randomness versus uncertainty, cost benefit analysis, etc. As mentioned, our concern is with the extension of the reliability analysis from the level of the individual member or component to that of the system as a whole. We will encounter each of these many elements in the discussion of systems reliability.

Within our limited context, however, we will focus on those elements only to the degree that they may be different or extended when considering the system as distinct from the component. Both the need for and limitations of current structural systems reliability as applied to the offshore industry have been discussed by several authors and participants in the "DIRT" Conference (*NRC, 1983*). We shall expand upon and extend that documentation.

Chapter 2

AN EXPLICIT FORMULATION OF STRUCTURAL SYSTEMS RELIABILITY, REDUNDANCY, AND ROBUSTNESS

With the several objectives of (1) showing the relationships among reliability elements in current practice, (2) clarifying precisely the distinctions between member reliability and systems reliability, and (3) demonstrating the potential role of systems reliability in the offshore industry, we turn next to a new, simple, yet rather complete presentation of structural systems reliability. The formulation is intentionally expanded in terms of a sequence of several identifiable and interesting factors. The purpose now is not how to calculate these terms, but rather their interpretation and utility (if they could be made available efficiently enough in practice). We shall return to numerical examples and to their evaluation in Chapter 3.

Eq. 2.1 represents the probability of failure of an offshore structural system during its proposed economic life:

$$\begin{aligned} p_{f_{SYS}} &\approx p_{f_{OL}} + p_{f_{EX}} \\ &\approx p_{DL} \cdot L \cdot O \cdot D_0 \cdot C_0 \cdot R_0 \cdot M_S \\ &\quad + \sum_i \sum_j p_{ij}\, L \begin{cases} 1 \cdots \textit{if system cannot survive without member } j \\ (p_{f_{damaged,ij}} + p_{f_{repair,ij}}) \cdots \textit{otherwise} \end{cases} \end{aligned} \tag{2.1}$$

The terms will be defined momentarily. Because we wish to introduce immediately in this report some precision, clarity and specificity, we shall use this equation as a vehicle and unifying theme. To function in this way, the equation should identify clearly the role of systems reliability in the offshore structural reliability problem. As expanded below, the equation addresses not only extreme load conditions, but also accidents, inspection, repair, residual strength, etc. It could be incorporated easily within a larger, full-scope systems analysis. Coupled with cost information and parameter variations, it can be used in the process of optimizing design, inspection, and repair, whether during design or operation, whether before or after an incident such as a boat collision affecting the structure.

I shall discuss the individual factors relatively informally here. They are defined more carefully in Appendix A, where many of the comments and observations in the report are collected for ease of reference. With only a few minor approximations (e.g., assuming small probabilities, we are able to ignore the probability of two or more major accidents during the life of the structure and to replace terms of order $(1-p_{f_{OL}})$ by 1) the formula gives a rather complete specification of the probability of failure of a structure during its operational lifetime. In the first line, the term $p_{f_{OL}}$ represents the probability of failure due to potential overloads and the second term, $p_{f_{EX}}$, represents the probability of failure due to exogenous events such as collisions, dropped objects, construction errors (e.g., improper welds), corrosion, and perhaps even fatigue failure, as will be discussed more fully below.

Member-Level Factors. The expression for the probability of failure due to overloads is made up of seven factors (see Fig. 2.1). The first, p_{DL}, represents the annual probability of (design) overload. In current practice this might be 10^{-2}. In a working stress-based code, it also represents the annual probability of overstress, i.e., the annual probability that the dominant external load, e.g., the wave load, will exceed the code capacity of a fully-stressed member (with proper allowance for dead, current, and wind load effects). This probability is, most precisely, the probability that the design wave height will be exceeded in a given year, but in keeping with what follows we prefer to think of it here as the probability that the nominal capacity of a typically fully-code-loaded member is exceeded by the (dominant) load. (We shall use the term "member" here in a generic sense to cover any individual element, e.g., a joint or pile, as well. Also, if more than one dominant load is of interest, e.g., multiple wave directions, a simple additive expansion is possible.)

The second factor, L, is the *economic life* of the structure, e.g., 20 years. Therefore the product of the first two terms represents, approximately, the probability that the dominant external load will exceed its design value (or the code capacity of a typical fully loaded structural member) in the life of the system. The product $p_{DL} \cdot L$ might be 0.2.

The third factor in Eq. 2.1, O, represents the *overload capacity or reserve member strength* implied in the current code. It is defined such that the product of the first three terms represents the lifetime probability that a typical fully-utilized (under code design) member will have its true capacity exceeded by the combination of the dead and applied external loads. The value of O will be significantly less than 1, perhaps in the order of 10^{-2}. If the code basis is working stress, the factor O represents primarily the difference between the allowable working stress and the yield stress. In general, however, it includes any dead-load and live-load factors, strength-reduction factors, the distinction between elastic and plastic section moduli, conservatisms in the prediction of the behavior of beam-columns, axially-loaded compression struts, etc. It is well known from simple member reliability analysis that the value of O depends strongly on the coefficient of variation of the wave load, implying that it will differ between the Gulf of Mexico and the North Sea, for example. These first three factors, $p_{DL} \cdot L \cdot O$ represent the topics discussed and analyzed in current member-level reliability theory and practice, as represented, for example, in the current API development of a probability-based LRFD design code (*Moses, 1986*). In fact, their product represents the probability associated with the code-accepted lifetime safety index, β. For β=3, for example, $p_{DL} \cdot L \cdot O$ is approximately 10^{-3}.

The next factor, D_0, modifies the formulation's member basis away from the code (as represented in $p_{DL} \cdot L \cdot O$) to the actual ***most-likely-to-fail (MLTF) member*** in the specific structure at hand. If the designer's philosophy of member-sizing has been more conservative than the code, D_0 will be less than 1, perhaps 10^{-1}. In a case study that will be discussed in Chapter 4, the value of $p_{DL} \cdot L \cdot O \cdot D_0$, (which is simply the largest lifetime failure probability among all individual members) was found to be less than 10^{-4} for an X-braced structure located in the Gulf of Mexico, implying a D_0 of less than 10^{-1} (if the current code basis is indeed 10^{-3}). On the other hand, D_0 may be greater than unity if one is re-evaluating an operating structure designed to an earlier code or being re-evaluated for a proposed increase in the operating deck loads.

Systems-Level Factors. These first four factors in Eq. 2.1 represent the as-practice-state-of-the-art: member-level reliability analysis. The next three factors represent the contributions associated with our focus: structural systems reliability. The first of these, C_0, represents *complexity*. It is defined (see Appendix A) such that the product of the first five terms ($p_{DL} \cdot L \cdot O \cdot D_0 \cdot C_0$) represents the probability that at least one structural member will fail (be loaded in excess of its capacity) in the life of the system. The value of C_0 depends on the number of members in the system or, more generally, on the number of potential failure initiators, including, for example, the foundation. (We shall continue to use the term "member" in this generic sense for simplicity.) In fact, a simple (upper bound) approximation for $p_{DL} \cdot L \cdot O \cdot D_0 \cdot C_0$ is the sum of the member failure probabilities. The value of C_0 is always equal to or greater than one (i.e., C_0 is a "negative" factor.) Although the complexity factor will be higher if there is a larger number of members, it also depends strongly on how many of these members are heavily stressed under the dominant loading condition. One would not expect to see, for example, the lightly wave-stressed members in the horizontal bracing system in a typical jacket platform contributing significantly to the complexity factor C_0. It has been observed by Moses (*1983*) that the application of formal member-level optimization will tend to increase what we call C_0. More subtly, we shall see that C_0 depends on the degree of (stochastic) dependence or correlation among the individual members' failures; this dependence is in turn strongly influenced by the relative uncertainty of the load with respect to the capacity. Notice that C_0 does not distinguish between a statically determinate and statically indeterminate structure. Also, up to this point the structural analysis involved is associated only with the original structure; post-failure behavior of members is not yet an issue. A design implication of this system-related factor is that: to the degree that avoidance of *any* member failure is a concern, and to the degree that C_0 grows with the number of members, all else being equal, a structure with more members should be designed with lower member-level failure probabilities.

Eq. 2.1's next factor, R_0, represents the ***redundancy*** of the intact system with respect to overloads. As shown in Appendix A, R_0 is defined such that the product of the first six factors will yield the lifetime probability of failure of the structural system with respect to overload. This factor R_0 will be less than or equal to one, i.e., a "positive" factor. It will equal one if the structure is statically determinate, i.e., if first-member failure implies system failure. It may also be one if the first-member failure leads to significant unloading of that member, load shedding to neighboring members, followed by their failure, and subsequent "unzipping" of the entire system. Given adequate member ductility, adequate residual system stiffness and/or adequate "backup" capacity in structurally parallel members, however, an indeterminate system will demonstrate a redundancy factor R_0 less than one*, implying system capacity beyond first-member failure. Although this factor R_0 includes this post-first-member-failure system capacity, (or system residual strength), it also must allow for the (possibly many) different potential sequences of member failures or modes of failure following one (of possibly many) different potential first-member failures. (These multiple possible sequences will be shown graphically below as a branching tree;

*It might be preferable to define both the complexity and redundancy factors as the reciprocals of C_0 and R_0 to give them more direct connotations, i.e., greater than unity is beneficial.

see Fig. 3.4.) It is, of course, the purpose of structural systems reliability analysis to calculate this probability, and we shall go into more details about just that in subsequent sections.

The final, seventh factor, M_S, in $p_{f_{OL}}$ stands for *systems model*. It is included simply for practicality. It is suggested here that, given current practice, it would be desirable today to calculate the first six factors discussed above by using only a conventional static pushover representation of nonlinear structural system behavior. If this suggestion is adopted for convenience, then the final factor M_S is necessary to correct the preceding factors to account for the relationship between the true system's failure behavior and that predicted by a simple static pushover analysis. For example, a structure subjected to seismic loads or dynamic wave loads or even static but repeated high wave loads would undergo a much more complex failure process than that modeled by the static pushover analysis. Issues such as system ductility, cumulative deformation and shakedown, and dynamic effects can only be represented through more complex structural-mechanical models, and hence, more complex structural systems reliability analysis. This factor M_S should also include any correction for possible simplifications in the failure criteria adopted in the static pushover systems reliability analysis, e.g., deflection and instability criteria. We shall demonstrate in subsequent chapters that it is possible, but perhaps not yet practically efficient, to conduct a systems reliability analysis including the last factor. As we shall see in subsequent sections and examples, systems reliability analysis as represented by the first six factors does appear to be economical and practical today.

Focusing for now on the first factors, $p_{DL} \cdot L \cdot O \cdot D_0 \cdot C_0 \cdot R_0$, we recall that the first four represent the conventional element-based reliability features and the latter two, C_0 and R_0, represent the systems effects. Vis-a-vis a simple member, a system is both more complex and possibly redundant. Hence our preference to separate these last two factors as indicated. It may, however, be the product of the two that is of ultimate interest. In one of the examples which will follow in Chapter 4, we will focus upon the difference in this product between the case of an X-braced and the case of a K-braced, large, three-dimensional, steel jacket structure. That case study found that the X-braced structure had a factor $C_0 \cdot R_0$ that was one to two orders of magnitude less (better) than that of the K-braced structure.

Exogenous Failures and Damaged "Members": In order to appreciate the role of systems reliability in such questions as damaged structures, inspection, repair, etc., let us turn to the second major term in Eq. 2.1, the probability of failure associated with (typically) exogenously initiated failures, $p_{f_{EX}}$. The term is expressed as a double summation, the first summation being over a set of scenarios, the second over a set of failed members. These scenarios include such accidents as boat collisions of various levels of severity, heavy objects dropped from the deck, riser-failure impacts, construction and design errors (e.g., bad connection weldments), and even quite possibly fatigue and fracture failures. The first two factors represent the lifetime probability of a particular scenario, i, leaving a particular member, j, failed. The first of these two factors, p_{ij}, represents the annual probability of the joint occurrence of scenario of type i and the event that member j is left damaged following that scenario. (The expression "member j" represents here a single member or, potentially, a set of two or more damaged members, joints, etc. The index j may also be over two or more levels of damage, e.g., a bent or ruptured tubular member or levels of local damage to a TLP hull.) The second factor, L, is again the life of the structure.

If the structural system cannot survive the damage of member j, then the (conditional) probability of system failure is unity, as indicated on the upper line* following the bracket in Eq. 2.1. If it can survive, then we must consider not only the possibility that the damage is undetected and the damaged structure fails under environmental loads prior to repair, but also the possibility that the damage is detected and repaired, and the structure subsequently fails anyway. These two possibilities are indicated in Eq. 2.1. Both can be expanded to display interesting issues. Again for illustration and precision, let us explore briefly just the former, damaged term, $p_{f_{damaged,ij}}$. The latter, "weakened by repair" term $p_{f_{repair,ij}}$, is somewhat similar to the intact structure with allowance for the fact that the damaged member is first detected and then repaired to (perhaps) somewhat different reliability than the original undamaged member (see Appendix A, Eq. A.3).

*A neater if less readable form of the second line of Eq. 2.1 is

$$\sum_i \sum_j p_{ij}\, L \left\{ \delta_j + (1-\delta_j)(p_{f_{damaged,ij}} + p_{f_{repair,ij}}) \right\}$$

in which δ_j is 1 if the system fails immediately upon sustaining damage level j , and zero otherwise.

The probability $p_{f_{damaged,ij}}$ can be written as

$$p_{f_{damaged,ij}} = (1-I_{ij}^{Q})p_{DL} \cdot O \cdot \overline{T}_j \cdot \overline{D}_j \cdot \overline{C}_j \cdot \overline{R}_j \cdot M_S \qquad (2.2)$$

The first factor, $1-I_{ij}^{Q}$, is the probability that the damaged member, j, is not detected in the post-accident (i.e., post-scenario i) inspection. The probability I_{ij}^{Q} is clearly a function of the *quality of post-accident inspection*. We would expect it to depend on the nature of the scenario and on the location (accessibility) and nature of damage to the member j. The value of I_{ij}^{Q} will clearly be unity for many kinds of important accidents.

The subsequent seven terms in Eq. 2.2 mirror those for the intact structure (Eq. 2.1). Here they must be modified to determine the probability of the failure of the structure in its damaged state due to the subsequent environmental conditions. We choose to introduce once again p_{DL} and O in order to reference the subsequent factors to this code-based member. Recall that the product $p_{DL} \cdot O$ represents the annual probability of failure of a hypothetical, fully-stressed-to-code member in the structure. We shall come back to the exposure time, $\overline{T}_j$. Like D_0, the factor $\overline{D}_j$ corrects from the code basis to the actual stresses in the MLTF (most-likely-to-fail) member. Now it must reflect the fact that the member stresses under any load, e.g., the design load, may very well be modified in the remaining members due to the reduced effectiveness or even absence of the failed member j.

If the failed member j was not critical to the stresses in the system (e.g., a redundant, horizontal bracing member in a jacket), $\overline{D}_j$ may still equal D_0. In general, however, owing to expected stress increases near the damaged member, it may be as large as 10 to 100 times D_0. The three factors $p_{DL} \cdot O \cdot \overline{D}_j$ represent the annual failure probability of the MLTF member in the damaged system. They capture the member-level problem.

The factor $\overline{C}_j$ in Eq. 2.2, as before, describes system complexity and, as before, represents the fact that there are many member failures that might potentially initiate the system failure under the applied environmental loads. We must expect the value of $\overline{C}_j$ to be different from that of the complexity factor C_0 in the intact structure, however. In fact, we might expect $\overline{C}_j$ to be better (smaller, closer to one) than in the intact structure. If the structure had been "well designed," there may be many members at comparably high stress levels in the intact structure, i.e., C_0 may be relatively high (10 or more). In contrast, in many cases, the damage of an important member may affect and increase substantially the stresses only in a local region. This creates, in effect, a "hot spot" among the members in the region of the failed member but not elsewhere. (This effect was observed in the case study to be described in Chapter 4.) The gross stress increase has been captured in $\overline{D}_j$. The spatial localization is contained in $\overline{C}_j$. This localization of heavily stressed members may be an important "damage mitigation" characteristic of certain structures that can only be identified through a systems analysis.

The factor $\overline{R}_j$ is again a system redundancy factor. Now it refers to the structure with a member damaged, hence the subscript j. As before the redundancy factor for the structure will depend upon such questions as member ductilities, structural stiffness of the system, and residual strengths of remaining members given the failure of yet another member in the system. Because, as we shall see, the redundancy factor is better (lower) if there is reserve capacity available in members in parallel load paths, it too may become better in the damaged structure than in the intact structure, again due to localization of the highest stresses.

The factor $\overline{T}_j$ represents the (expected) time of exposure of the damaged structure. It can be expressed as

$$\overline{T}_j = \frac{T}{2} I_{oj}^Q + \frac{3T}{2}(1-I_{oj}^Q) I_{oj,2}^Q + \frac{5T}{2}(1-I_{oj}^Q)(1-I_{oj,2}^Q) I_{oj,3}^Q + \cdots \qquad (\overline{T}_j \leq L/2)^* \quad (2.3)$$

$\overline{T}_j$ is thus a function of the normal interval between regular inspections, T, and the effectiveness of quality of those inspections. The term I_{oj}^Q represents the probability of detecting the damaged member j during a typical inspection. The possibility also remains that the first regular inspection will fail to find it. The possibilities are indicated by the second, third, and potentially long series of such terms in Eq. 2.3.† The factor one-half, as in $T/2$, appears because in expectation the accident will occur with one half of the regular inspection interval remaining.

Finally, the factor M_S in Eq. 2.2 is introduced as before to represent any systems reliability modeling errors. In particular, should the preceding factors all be based on a simple static pushover model, the factor M_S would attempt to correct the failure model to a more realistic one involving repeated and/or

*If all I's are zero, $\overline{T}_j = L/2$.

†Additional subscripts, such as $I_{oj,2}^Q$ indicate that the probability of detection at the second regular inspection is conditional on not finding the damage under previous inspections. This number may be lower than I_{oj}^Q.

dynamic load effects.

Standing back and taking some distance from this long series of terms representing the probability of failure associated with exogenous accidents (Eqs. 2.2 and 2.3), we recognize several interesting features. The inspection policy is represented through the inspection interval T and the inspection quality factors, e.g., I_{oj}^{Q} . One interesting policy/system reliability interaction is the potential to trade-off inspection cost versus system robustness with respect to potential damage. The latter is represented by the terms to the right of the brace in Eq. 2.1. One can envision allocating inspection resources to each different member in proportion to the relative sensitivity of the system to that member's failure, as measured by the value of its term to the right of the brace, as well as its susceptibility to damage, p_{ij}.

Provided that the system does not fail immediately upon the accident, the most interesting part of Eq. 2.2, for our purposes, is the series of seven factors, p_{DL} through M_S. Comparing them with the corresponding factors in Eq. 2.1 for the overload failure of an intact structure, we see several interesting similarities and contrasts. By our construction of the formulation here, the common factors $p_{DL} \cdot 0$ represent the probability of failure of a typical major member in an intact structure designed to the code. Again, this is the typical focus of a member-based reliability analysis or design code. In the damaged structure this probability is enhanced (typically) by the factor $\bar{D}_j$; the ratio $\bar{D}_j / D_0$ reflects the increased stresses per unit load in surviving members of a damaged structure. The complexity factor $\bar{C}_j$ is to be contrasted with the factor C_0 in the intact structure; as discussed above, the ratio $\bar{C}_j / C_0$ may well be less than one due to the typical localization of high stresses in the damaged structure. The redundancy factor, $\bar{R}_j$, will now vary from member to member in the structure, reflecting thereby the relative residual redundancy of the system. (These factors, $\bar{C}_j$ and $\bar{R}_j$, are undefined for those members that lead to immediate system failure).

The product of the factors $\bar{D}_j \cdot \bar{C}_j \cdot \bar{R}_j$ reflect the dependencies of the residual reliability on j, the member damaged. We shall see in a case study of a large steel jacket structure, to be discussed more fully below, that the product of these three terms for a critical member within the vertical bracing systems of that structure may have a value two orders of magnitude larger than the corresponding factors, namely, $D_0 \cdot C_0 \cdot R_0$, for the intact structure.

Robustness: It may well be that the most important application of systems reliability is in the evaluation of the ***robustness*** of a design. By this expression we mean here its ability to sustain exogenous damage with a limited loss of reliability. The ratio $\overline{D}_j \cdot \overline{C}_j \cdot \overline{R}_j / D_0 C_0 R_0$ is a useful measure of the system's robustness with respect to damage to a particular "member" j . (Again, the larger the value the worse the robustness.) System-wide robustness is particularly important in comparing novel designs with current offshore structural systems. In order to evaluate robustness quantitatively at the time of design one must "average" the reliability (vis-a-vis the intact structure) over the members, $j=1, \cdots, n$. The simplest such measure is

$$ROB_1 = \frac{\frac{1}{n}\sum_{j=1}^{n} \overline{D}_j \cdot \overline{C}_j \cdot \overline{R}_j}{D_0 \cdot C_0 \cdot R_0} \tag{2.4}$$

obtained by dividing typical (annual) terms in Eq. 2.2 by the (annual) intact structure term $p_{DL} \cdot O \cdot D_0 \cdot C_0 \cdot R_0 \cdot M_S$ in Eq. 2.1., and averaging. (For notational simplicity we do not treat the members that cause immediate failure differently; they should in fact be entered in the sum as $1/(p_{DL} \cdot O \cdot M_S)$ for consistency.) For the perfectly robust system, there is no reliability deterioration no matter which member fails and $ROB_1=1$. For a statically determinate system the robustness is minimal, and (considering only complete member failures as damage states) $ROB_1=1/(p_{DL} \cdot O \cdot D_0 \cdot C_0 \cdot R_0 \cdot M_S)$, i.e., the reciprocal of the annual intact-system failure probability, a number such as 10^4.

This simple robustness measure implicitly assumes all members are equally likely to be damaged. A more complete robustness measure would allow for both the relative likelihood of damage and non-detection of different members:

$$ROB_2 = \frac{\sum_i \sum_j p_{ij} L \left[(1-I_{ij}^Q) p_{DL} \cdot O \cdot \overline{T}_j \cdot \overline{D}_j \cdot \overline{C}_j \cdot \overline{R}_j \cdot M_S \text{ or } 1 \right]}{\sum_i \sum_j p_{ij} L \, (1-I_{ij}^Q) p_{DL} \cdot O \cdot L \cdot D_0 \cdot C_0 \cdot R_0 \cdot M_S} \tag{2.5}$$

the measure with respect to the individual relative likelihood factors $p_{ij}(1-I_{ij}^Q)$. (Again allowance should be made for members causing failure by the term "or 1" in Eq. 2.5. If no such "members" or damage cases exist several factors, e.g., L, p_{DL}, O , and M_S will cancel out of the definition.) These measures deserve more careful study at a later time. In private communication, it has been suggested by Prof. T. Moan that recommendations be developed for member p_{ij} and I_{ij}^Q values as a function, say, of vertical location; collision scenarios would have relatively high levels near the waterline, inspection quality, I^Q, would deteriorate with depth. There may also be a region of relatively low crack inspection quality in the splash zone.

It can be seen in Eq. 2.5 that inspection and repair will potentially improve the apparent damaged structure reliability loss, $\overline{D}_j \cdot \overline{C}_j \cdot \overline{R}_j / D_0 \cdot C_0 \cdot R_0$. This benefit is reflected by the fact that the ratio of $\overline{T}_j$ to L may be in the order of 10^{-1}, i.e., the exposure time of the damaged structure prior to inspection and repair may be small relative to the entire life of the structure. We can see, too, the interaction between relative inspection effectiveness and the relative importance of the member to the structural system as a whole; one might like the product of the non-detection probability, $(1-I_{ij}^{Q})$, and $\overline{D}_j \cdot \overline{C}_j \cdot \overline{R}_j$ to remain relatively constant for all members in the structure. If the member is important to the system, as for example a compression member in an X-brace of a vertical bracing system in a steel jacket platform, one would expect the $\overline{D}_j \cdot \overline{C}_j \cdot \overline{R}_j$ factor to be high, and one would therefore want the inspection to be of high quality, and/or the inspection interval, T, short, so that $(1-I_{ij}^{Q})\overline{T}_j$ is near zero. Less important members would be reflected in low values of $\overline{D}_j \cdot \overline{C}_j \cdot \overline{R}_j$ and could as a result be permitted higher values of $1-I_{ij}^{Q}$ and T.

Fatigue. At the moment it appears that fatigue can for convenience be treated in the context of this model as if it were an exogenous event. It has many of the same characteristics: it is likely to be an undetected welding or other construction flaw that serves as the initiator, inspection may help identify and repair the growing crack, the failure event is likely to occur at less-than-extreme loads, failure of one member may increase stresses elsewhere inducing immediate system failure or subsequent overload failures of the weakened system, etc. These characteristics can be accommodated in this model.

The time-varying cumulative or growing nature of fatigue may suggest a special treatment in some cases, however. Fortunately, current member-level evidence suggests that in many cases the coupling of fatigue and overload capacity (e.g., a fatigue crack weakening a member with respect to ultimate capacity) can be neglected in reliability analysis; because of the very rapid acceleration in the later stages of crack growth, the interval where this weakening effect might exist to any significant degree is very short compared to the earlier life of the structure (e.g., *Guers and Rackwitz, 1986*). The structural-system-related issue of the possible relaxation of stresses (and hence reduced growth rates) in a deteriorating member within a statically indeterminant structure can, from a reliability point of view, be handled at the component level. From the systems reliability perspective, one can still imagine special issues: for example, there is dependence among different members' crack growth rates due to common load histories (a presumably small effect, however, because of the relatively small contribution of load randomness in the fatigue problem). The possibility of a sequence over time of (only) fatigue-induced component failures leading to system failure is a new problem that is current investigation at the Technical University of Munich. More such work is recommended.

Conclusions. Finally, to define the primary purposes of this report, we can look again upon all the terms in $p_{f_{SYS}}$ (Eqs. 2.1-2.3). We recognize that most of its elements are common to member-based reliability analysis. Some are related to scenario (accident) likelihoods and several are related to inspection quality. Indeed, the only factors that strongly reflect systems reliability analysis needs are the C and R factors: C_0, $\tilde{C}_j$, R_0 and $\overline{R}_j$. As mentioned, they reflect the complexity and redundancy of the intact structure and of the residual structure, given damage to member j.

In subsequent sections we shall discuss how systems reliability may be used to evaluate such factors and we will present some of the case study results in Chapter 4 in terms of such factors in order to maintain a continuity through the report. The author does not contend that this model of overload and accident/inspection reliability is perfect but it should be adequate for the purposes of this report to serve the function of defining more explicitly the particular role of systems reliability in overload and accident safety of offshore structures. In particular, the model may as stated need reinterpretation for structures such as semi-submersibles and compliant structures whose construction may include continuous hulls and line-like elements instead of or in addition to the linear structural members of the type envisioned in the discussion above, which was motivated primarily by steel jacket platforms.

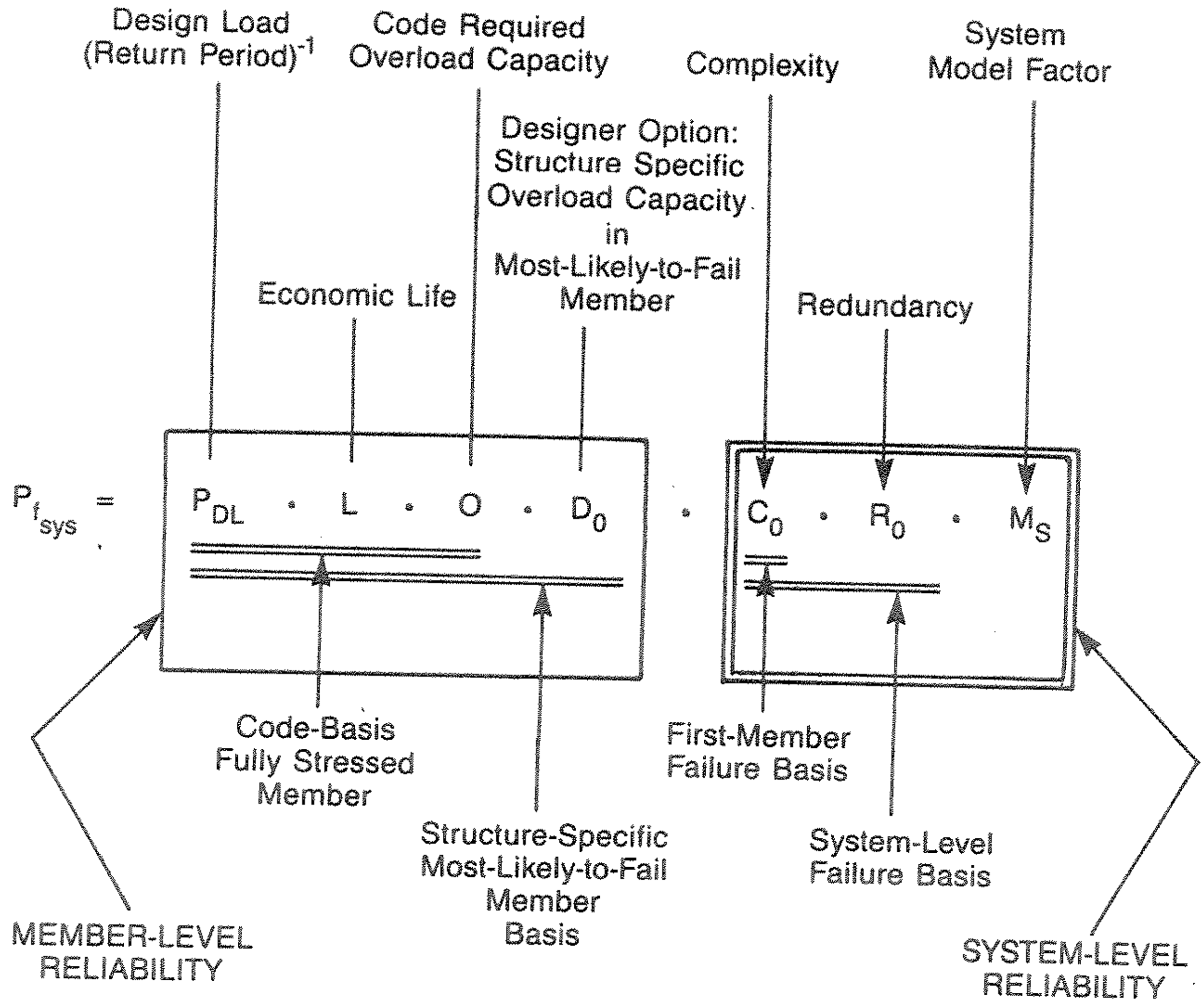

Fig. 2.1: Structural System Reliability Formulation:
Extreme Load, Intact Structure.

Chapter 3

ELEMENTS OF STRUCTURAL SYSTEMS RELIABILITY ANALYSIS

In this chapter we shall give an introduction to the analysis of structural systems reliability. We focus not on how to do the numerical calculations but rather on the formulation of the systems problem in terms (1) of the probabilistic representations of member-failure events and (2) of relevant random variables such as loads, member capacities, and post-failure member forces. We shall also develop expressions for the factors C_0 and R_0 that we have identified in the preceding chapter to be particularly important to the systems-level reliability evaluation of the structure (as distinct from the member-level evaluation). We can then deduce logically the general dependence of these quantitative measures of system complexity and redundancy on important factors such as relative load-to-resistance variability, degree of brittleness, etc. Finally, we shall display some numerical results for C_0 and R_0 in some ideal structural systems; the results show clearly the same quantitative effects that are found in real problems as well. A thorough understanding of all these sections will leave an engineer with a stronger ability to anticipate the role of the system level in the reliability of his own structure.

3.1 Determinate or Series Systems

Consider first a simple statically determinate structural system such as that shown in Fig. 3.1a. It has random member capacities R_i and a load pattern scaled by the random variable S. It can be replaced for analysis purposes by an even simpler system as shown in Fig. 3.1b. In that system representation, the chain-like nature of the statically determinate structure is emphasized. It becomes clear that the system will fail if its weakest link fails, e.g., if member 1 or if member 2 is overloaded*. Generalizing, we can write the ***system failure event*** F_s for a statically determinate structure as the union of the ***member failure events***, $F_i = \{S > \tilde{R}_i\} = \{\tilde{R}_i - S < 0\}$, with S and $\tilde{R}_i$ being the load and member capacity* random variables indicated:

*As shown in Fig. 3.1, one should in principle distinguish R_i, the member capacity defined in member force terms and $\tilde{R}_i$, the member capacity defined in applied load terms, where $\tilde{R}_i = R_i / c_i$. In this section it is somewhat neater to work with the latter. In the next section we find R_i more convenient.

The probability of system failure, $p_{f_{SYS}}$, can then be expanded into intersections of member failure events:

$$p_{f_{SYS}} = P[F_1 \cup F_2 \cup \cdots \cup F_n]$$

$$= \sum_{i=1}^{n} P[F_i] - \sum_{i} \sum_{j>i} P[F_i \cap F_j] + \sum_{i} \sum_{j>i} \sum_{k>j} P[F_i \cap F_j \cap F_k] - \cdots \tag{3.2}$$

The member failure event probabilities, $P[F_i]$, can be represented as $P[S > \tilde{R}_i]$ or $P[\tilde{R}_i - S < 0]$ or $P[g(\tilde{R}_i, S) < 0]$, in which the last form emphasizes that the member failure probability is associated with the negativity of a function of random variables (more generally, $\tilde{R}_i$ and S will typically themselves be functions of more ***basic random variables***, $\underline{X}$). The intersection events such as $F_1 \cap F_2$ represent the event that the capacities of both members are less than the applied load, i.e., that two functions, $g_1(\tilde{R}_1, S)$ and $g_2(\tilde{R}_2, S)$ are both less than zero.

Consistent with our formulation in the preceding section, we choose to divide through the right-hand-side of this equation by the failure probability $P[F_1] = P[S > \tilde{R}_i]$, which we choose (simply by proper numbering) to be the largest value in the set of member failure probabilities:

$$p_{f_{SYS}} = P[F_1] \left(1 + \sum_{i=2}^{n} \frac{P[F_i]}{P[F_1]} - \sum_{i} \sum_{j>i} \frac{P[F_i \cap F_j]}{P[F_1]} + \cdots \right) \tag{3.3}$$

in which $P[F_1] = \max_i (P[F_i])$.

We recognize that the leading factor $P[F_1]$ is simply what we called in the preceding section the probability of failure of the most-likely-to-fail (MLTF) member or $p_{DL} \cdot L \cdot O \cdot D_0$ (see Eq. 2.1). We also recognize that because the structure is statically determinate, the system is certain to fail if any member should fail. Therefore, the factor R_0 in the preceding section's Eq. 2.1 is unity. Also, given the ideal structure, the factor M_S for systems modeling is unity. We conclude that C_0, the complexity factor, is equivalent to the second factor, that in parentheses, in Eq. 3.3 above.

For the statically determinate structural system, our analysis of the system reliability factors C_0 and R_0 reduces to the evaluation of the complexity factor

$$C_0 = 1 + \sum_{i=2}^{n} \frac{P[F_i]}{P[F_1]} - \sum_{i} \sum_{j>i} \frac{P[F_i \cap F_j]}{P[F_1]} + \cdots \tag{3.4a}$$

In many cases, the most interesting term is the second, the sum of the ratios of the member failure probabilities divided by the maximum member failure probability. As designed and constructed, many structures find a large fraction of their members not stressed to allowable capacity or to comparable levels. In probabilistic terms the implication is that such a member's failure probability is an order of magnitude or more less than that of the most heavily stressed member in the structure (i.e., the MLTF member). The implication here is that many members will not contribute significantly to the summation, or, hence, to C_0. In contrast, in the "worst" case, all members will be as likely to fail as the MLTF member. In this case the second term in Eq. 3.4 will be equal simply to $n-1$, where n is the number of members in the structure.

The third term in Eq. 3.4 for C_0 involves a double summation of the probabilities of the intersections of two member failure events, i.e., $P[F_i \cap F_j]$, which is read as "the probability of the overload of member i <u>and</u> the overload of member j" (implicitly, in the intact structure). These intersections represent very explicitly *systems* effects. Their probabilities are much more difficult to calculate, but we will not go into computational details in this chapter. Suffice it here to consider two limiting cases.

Independent Member-Failure Events. In the first case, the failure events F_i and F_j are assumed to be probabilistically independent. This will be the case in practical situations if the failure events are dominated by the randomness in the capacities of the members (rather than by that of the loading, i.e., if the COV of the capacity is large relative to that of the load) and if those capacities are in turn independent random variables. In these circumstances, the numerator in the third term can be replaced by the product of individual failure probabilities, i.e., $P[F_i]\cdot P[F_j]$. In practical reliability problems both of these factors in the product are small, e.g., in the order of 10^{-3}. Therefore we expect these terms in the (negative - signed) double summation and the summation itself to be of small order and negligible compared to unity. By a similar argument, we can ignore all of the terms in the triple and quadruple sums, etc. in the equation, provided all failure events are mutually independent. We conclude that in this case of independent failure events, the complexity factor C_0 is approximately

$$C_0 \approx 1 + \sum_{i=2}^{n} \frac{P[F_i]}{P[F_1]} \leq n \qquad (3.4b)$$

The effective upper bound, $C_0 = n$, reflects the worst case in which all members are equally likely to fail.

Perfectly Dependent Member-Failure Events. The second limiting case is when the member failure events are perfectly probabilistically dependent. In general, strong failure event dependency arises because, for example, there is a common load random variable in all of the member failure events, $F_i \cap F_j = [\{S > \tilde{R}_i\} \cap \{S > \tilde{R}_j\}]$. Given that member 1 has failed, it is much more likely that member 2 has failed simply because the implication of the failure of 1 is that the load is probably very large. The case of perfect dependence arises if the load is the only random variable (or if $\tilde{R}_i$ and $\tilde{R}_j$ are perfectly correlated). In this case, if member 1 is the weakest member, then the conditional probability that member 1 will fail given that member 2 has failed is clearly one. Therefore, the probability of the joint event failure of 1 and 2 is just the probability of the failure of 2. Continuing this argument and collecting all the terms in the multiple summations, one finds that these terms cancel out not only one another but also those terms left in the single summation in Eq. 3.4, leaving finally $C_0 = 1$. In the footnote*, a somewhat more direct proof of this conclusion is shown.

General Correlated Case. In general, if neither independence nor perfect dependence of the member failure events exist (and this is of course the typical real situation) the value of C_0 will lie somewhere between these two limits. This is shown in Fig. 3.2. In the intermediate range, it can be shown by proper calculation (see numerical results to follow) that the value of C_0 will depend on such questions as: (1) the degree of dependence among the failure events (as measured by, say, the correlations among *safety margin* random variables, $M_i = R_i - S$, of the individual members), (2) the general level of the failure probabilities, (3) the relative values of the member failure probabilities, and (4) the type of probability distributions involved.

Perhaps the most important of these factors is the correlation between the member safety margins**: $\tilde{M}_i = \tilde{R}_i - S$. Both the capacity and the load are typical-

*$F_1 \cup F_2 \cup F_3 = F_1 \cup [F_1^c \cap (F_2 \cup F_3)]$ and F_1 is exclusive from the events in brackets. Therefore the probability of the union of failures is $P[F_1] + P[F_1^c \cap (F_2 \cup F_3)]$. In the perfectly dependent case, if the "weakest" member does not fail, i.e., F_1^c, then no other member will fail, so that $P[F_2 \cup F_3 | F_1^c]$ is zero. Hence the second term, which can be written $P[F_2 \cup F_3 | F_1^c] P[F_1^c]$ is zero. Conclusion: the probability of the union of failures is just the probability of the most likely failure $P[F_1]$ in the perfectly dependent case.

**As mentioned in the footnote on pg. 3-1, in the next section we shall prefer to work with member capacities R_i and hence safety margins $M_i = R_i - c_i S$. It can be shown easily that R_i and $\tilde{R}_i$, which equals R_i / c_i, have the same COV, that the correlation coefficient between R_i and R_j equals that between $\tilde{R}_i$ and $\tilde{R}_j$, and finally, somewhat less obviously, that the correlation coefficient between M_i and M_j equals that between $\tilde{M}_i$ and $\tilde{M}_j$. Therefore conclusions relative to safety margin correlations are generally independent of the capacity convention.

ly random variables. For the case (of practical interest) in which the standard deviations of the resistances are all the same, the correlation coefficient between any two member safety margins is:

$$\rho_{ij} = \frac{\bar{\rho}_{ij}\sigma_{\tilde{R}}^2 + \sigma_S^2}{\sigma_{\tilde{R}}^2 + \sigma_S^2} \tag{3.5}$$

In this equation, $\bar{\rho}_{ij}$ is the correlation coefficient between the resistances or capacities of any pair of members i and j in the system, $\sigma_{\tilde{R}}^2$ is the (common*) variance of the resistances, (measured, recall, in load terms) and σ_S^2 is the variance of the applied load. The left hand side, ρ_{ij}, is the correlation coefficient between the pair of member safety margins. This is an important practical case because, as we have seen above, only members with relatively large failure probabilities are of interest in the calculation of the system failure probability.

A convenient alternative to the safety margin ($M_i = \tilde{R}_i - S$) is the safety factor $\tilde{R}_i / S$ or its (natural) log $\ln\tilde{R}_i - \ln S$, and it produces a somewhat different measure of the dependence between member failures (overloads). Let us define $M_i^* = \ln\tilde{R}_i - \ln S$ (or equivalently $M_i^* = \ln R_i - \ln S - \ln c_i$, in member force capacity terms; recall $\tilde{R}_i = R_i / c_i$). Then a convenient approximation† of the correlation coefficient between any two (log) margins, M_i^* and M_j^* is

$$\rho_{ij}^* \approx \frac{\bar{\rho}_{ij} V_i V_j + V_S^2}{V_i V_j + V_S^2} \tag{3.6}$$

in which V_i is the coefficient of variation of the resistance (whether it is measured as R_i or $\tilde{R}_i$, i.e., in member force or applied load terms).

Both Eqs. 3.5 and 3.6 demonstrate that the correlation between safety margins, which we know affects the value of the complexity factor C_0, depends upon the level of the randomness in the loading (as measured by σ_S^2 or V_S^2) relative to the total randomness in the member safety margin uncertainty (which is

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*In general $\rho_{ij} = (\bar{\rho}_{ij}\sigma_{\tilde{R}_i}\sigma_{\tilde{R}_j} + \sigma_S^2) / \sqrt{\sigma_{\tilde{R}_i}^2 + \sigma_S^2}\sqrt{\sigma_{\tilde{R}_j}^2 + \sigma_S^2}$. In terms of resistances measured in member forces, R_i, the general result can be found by replacing each $\sigma_{\tilde{R}_i}$ by σ_{R_i} / c_i.

†The exact result is $\rho_{ij}^* = (\bar{\rho}_{ij}^*\sigma_i^*\sigma_j^* + \sigma_{\ln S}^2) / \sqrt{\sigma_i^{*2} + \sigma_{\ln S}^2}\sqrt{\sigma_j^{*2} + \sigma_{\ln S}^2}$ in which σ_i^* is the standard deviation of the log of R_i (or $\tilde{R}_i$) and $\bar{\rho}_{ij}^*$ is the correlation coefficient between $\ln R_i$ and $\ln R_j$. The approximation uses the fact that σ_i^* is approximately V_i, $\sigma_{\ln S}$ is approximately V_S, and $\bar{\rho}_{ij}^* \approx \bar{\rho}_{ij}$. The further approximation in the denominator is a simplification that makes the result nicely symmetrical for ease in presentation and recollection. Because of the approximate numerical equivalence between (1) the correlation coefficient between any two variables and (2) the correlation coefficient between their natural logs, ρ_{ij}^* is also approximately the correlation between the two safety factors $SF_i = \tilde{R}_i / S$ and $SF_j = \tilde{R}_j / S$, which gives it added significance.

reflected in the denominators). An additional factor which may enter is the correlation among the capacities of the members, $\bar{\rho}_{ij}$.

In many analyses in the literature and in practice, this capacity correlation factor is assumed to be zero. In that case, and assuming further that the coefficient of variation of the resistances is equal for all members, we can make the point about relative randomness very clearly. Eq. 3.6 becomes simply:

$$\rho_{ij}^{*} \approx \frac{V_S^2}{V_R^2 + V_S^2} \tag{3.7}$$

It states that the (approximate) correlation coefficient between two (log) safety margins in the structure is simply the *ratio* of the squared *COV* of the load to the sum of the squared *COVs* of the load and the typical member capacity. For example, if the coefficient of variation of the loading is 40% and the coefficient variation of the capacity if 10%, this correlation coefficient is 94%; if the coefficient of variation of the load and resistance are equal, the correlation coefficient is 50%.

The two correlation measures, ρ_{ij} and ρ_{ij}^{*}, clearly do not have the same numerical values in any given case. Typically they will both be "high" (or not) however. They both need to be presented, however, because both margins and log margins (and/or safety factors) are in common use in systems reliability. Analytically, one will obtain the same final system failure probability no matter which way he chooses to formulate the problem. These correlation coefficients are primarily simply helpful intermediate indicators of probabilistic dependence. As yet there is not enough numerical experience to ascertain the relative merits of the two measures. We shall subsequently refer to them both generically as margin correlations, ρ_{ij}.

It can be demonstrated (see Section 3.3 for illustrations) that the complexity factor C_0 is rather insensitive to the value of the correlation coefficient ρ_{ij} until that coefficient and/or the number of members becomes large. For example, if the number of members is only 2, this correlation coefficient may need to be 98 or 99% to have a significant impact upon the complexity factor at the low probabilities of interest in our problems. As the number of important members (i.e., those with relatively high failure probabilities) grows, the level of this correlation coefficient necessary to impact C_0 falls to lower levels, e.g., 90 or 80%. By "impacting" the complexity factor C_0 we mean that the value of C_0 might begin to differ significantly from that associated with the independence assumption. Recall, under this assumption the value of C_0 can be well approximated by 1 plus the single summation of normalized member failure probabilities in Eq. 3.4b).

Series System Conclusion. We conclude then in brief that for the statically determinate or "series" system, as it is sometimes called, the net system effect denoted $C_0 \cdot R_0$ in the preceding section is simplified to C_0 because R_0 is unity. C_0 in turn is approximately one plus the normalized sum of individual member failure probabilities (the sum excluding the MLTF member, Eq. 3.4b), unless correlation effects are significantly strong. In fact, these correlation effects may be strong in cases of practical interest in offshore structures application as the examples above (and to follow) suggest.

We should remember that the series model just discussed is applicable if system failure is *defined* for practical reasons to be simply first-member failure. In this case, the analysis above holds even if the structure itself is statically indeterminate. Such a definition might be appropriate in the analysis of platform availability rather than safety. We should also realize that the model holds with respect to other than member failures; more generally what has been referred to above as a "member" might be any failure mode, including foundation failure, a shear failure as opposed to bending failure.

3.2 Indeterminate Structural Systems

We turn now to a more complex structural behavior than the statically determinate system in Section 3.1. The simplest possible statically indeterminate model is that shown in Fig. 3.3, where the structural system can be represented by two members acting in parallel, as shown.

In the paragraphs which follow, we will first set up a formulation of the probability of failure of this indeterminate structure and similar such structural systems, and then try to describe what can be said about this probability for particular classes of structural behavior of the individual members. Following that, we shall present the results in terms of the formulation of Chapter 2, namely, in terms of C_0 and R_0, the complexity and redundancy factors. Again, we shall try to describe, at least qualitatively, how these factors behave, depending on the type of structural behavior and characteristics such as the correlation among structural safety margins, as we did in the preceding discussion of statically determinate systems. In the following section, we shall present a set of numerical results for some simple idealized parallel or redundant structural systems, in order to see numerically how factors C_0 and R_0 behave.

Failure Sequence Formulation: Member Failure Events. Consider our simple two-member parallel system. Suppose that it is adequate to represent each member as being either "intact" or "failed," i.e., binary. In this case, it is helpful to cast the description of the problem into a ***failure-sequence tree***, such as that shown in Fig. 3.4a. In words, the tree says that the intact structure, as denoted by the node in the far left, may fail either by Member 1 failing first and then Member 2 failing, or, alternatively, by Member 2 failing first, followed by Member 1. We will use this observation to create more formal statements below.

First recognize, however, that much more complex structural systems can be represented in this same convenient format, as shown in Fig. 3.4b. Suppose that we have a large, statically indeterminate frame. The first set of branches on the left represents possible first-failing Members 1 through N, let us say. The second set of nodes represents possible second members to fail in the sequence. For example, on the upper set of branches, if Member 1 fails first, it may be followed by either Member 2 or 3, or any member through Member N. One specific sequence, namely Member 1 followed by 2 followed by 3, is represented by the uppermost three branches on the tree. In that case, we assume that the system fails, after this third-member failure, as indicated by the double circle. Other parallel sequences, such as Member 1 followed by 2 followed by Member N, may not yet imply failure, and the set of branches representing this sequence may continue.

In this way, we can graphically display the possible sequences of member failure that will lead to system failure. Notice that it is possible, in principle, in this binary-member situation to enumerate all possible failure sequences. If, however, the system contains a large number of members and is highly redundant, there will be many possible initial branches and potentially many possible levels of branches. It is this exploding of the failure-sequence tree that makes it difficult, or rather expensive, to calculate the system failure probabilities of large structures by attempting to enumerate and evaluate all possible failure sequences. As we shall see in Chapter 6, one of the primary objectives of practical structural systems reliability is to constrain the search through this tree to a limited number of important or major probability-contributing sequences. Notice, incidentally, that the statically determinate structure discussed in the preceding paragraphs is associated with a tree with multiple branches from the first node, but no subsequent branches.

Systems reliability analysis consists of determining the probability of each of the possible member-failure sequences, and then, the probability of at least one of the sequences happening. This will be the system failure probability. In more formal terms, the probability of a sequence will be the probability of an intersection of events, e.g., Member 1 fails and Member 2 fails and Member 3 fails, whereas the system failure probability will be the probability of a union: Sequence 1 fails or Sequence 2 fails or Sequence 3 fails, etc. Again, note that for a statically determinate structure, or for a situation, such as availability analysis, in which system "failure" is defined as first-member failure, the problem simplifies to identifying the probability of occurrence of at least one of the branches emanating from the leftmost node. This was formally stated in eq. 3.1 in preceding paragraphs. We can anticipate that the complexity factor C_0 is related to this calculation.

Let us return, now, to the two-member system identified in Fig. 3.3b and Fig. 3.4a, and describe the events more formally:

$$F_{system} = (F_1^{first} \cap F_2^{(1)}) \cup (F_2^{first} \cap F_1^{(2)}) = F_{s_{12}} \cup F_{s_{21}}$$

$$= \bigcup_i F_{s_i} = \bigcup_i \bigcap_j F_{i,j}^{ordered} \tag{3.8}$$

This states that the event, "F_{system}" can be described as the union of the two events shown in parentheses. Each of these events is associated with a sequence of member failures, namely, 1 followed by 2 and 2 followed by 1. The first sequence, 1 followed by 2, can be written formally as $F_1^{first} \cap F_2^{(1)}$, where F_1^{first} is the event that Member 1 is the first to fail, i.e., it is weaker than Member 2 with respect to the applied load. $F_2^{(1)}$ indicates that Member 2 fails in the system that remains, once Member 1 has failed. Exactly how that damaged system behaves will, of course, depend upon whether Member 1 has failed in a brittle or ductile manner, as we shall discuss below. For the moment, these distinctions are not necessary.

The second event in parentheses is associated, again, with an intersection of two events, namely, the second member is the first to fail, and Member 1 fails in the structure associated with the post Member 2-failed condition. In Eq. 3.8, we next write this statement as $F_2^{first} \cap F_1^{(2)}$, or as F_{21}. The first form, the total equation, states that the failure of the system is the union of the event "failure sequence 1-2" and the event "failure sequence 2-1." The second form is simply a shorthand notation for that statement. In the last form of Eq. 3.8, we emphasize that the system failure event can be written as a union of intersections of member failure events, with proper ordering considerations.

Failure Sequence Formulation: Safety Margins. Before pursuing the evaluation of the probability of these events, let us express them in somewhat more detail by replacing each event, such as F_1^{first}, by its corresponding statement in terms of the safety margins of the structural members. We denote M_i^0 as the safety margin of Member i in the intact structure, i.e., resistance of Member i minus the load effect applied to Member i. In the simple two-member ideal system, this margin $M_i^0 = R_i - S/2$. We use the superscript "0" throughout to indicate that this is the safety margin of that member when the structure is in its original, intact condition. With this notation, we can write the first event in Eq. 3.8, i.e., F_1^{first}, as $[(M_1^0 < M_2^0) \cap (M_1^0 < 0)]$.* In words, this says that, assuming a monotonically increasing load, Member 1 will both fail and fail *before* Member 2 if M_1^0 is less zero and if the safety margin M_1^0 is less than the safety margin M_2^0. The second event we must consider is $F_2^{(1)}$. For this event, we must consider a new safety margin, namely, the safety margin of Member 2 in the partially deteriorated structure, i.e., the structure with Member 1 failed. We denote this margin as $M_2^{(1)}$. Exactly how the safety margin $M_2^{(1)}$ relates to capacities of the members and the loads will depend, in turn, upon whether Member 1 failed in a ductile or brittle manner. For example, if Member 1 was brittle, Member 2 must now take all the load; the safety margin $M_2^{(1)}$ becomes $R_2 - S$ (in contrast to M_2^0, which is $R_2 - S/2$). This sequence is shown in Eq. 3.9, where in addition we see the second failure sequence involving the failure of Member 2 first, followed by the failure of Member 1, all expressed in terms of safety margins.

$$F_{system} = \{ [(M_1^0 < M_2^0) \cap (M_1^0 < 0)] \cap (M_2^{(1)} < 0) \}$$
$$\{ \cup [(M_2^0 < M_1^0) \cap M_2^0 < 0] \cap (M_1^{(2)} < 0) \} \tag{3.9}$$

Safety Margins for Two-Member Semi-Brittle Ideal System. Other examples of member safety margins in the two-member systems are shown in Eqs. 3.10-3.12 for different types of post-failure behavior of Member 1:

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*This result and Eq. 3.9 hold for the two-member ideal system, Fig. 3.3b. For a general two-member system we must write $M_1^0 < (c_1/c_2) M_2^0$ for the ordering event, where c_1 and c_2 are the coefficients defined in Fig. 3.1a. For the ideal system $c_1 = c_2 = 1/2$, implying $c_1/c_2 = 1$. See Appendix B of the companion report by Karamchandani (*1987*) for derivations and generalizations.

Brittle:

$$M_1^0 = R_1 - S/2 \tag{3.10a}$$

$$M_2^{(1)} = R_2 - S \tag{3.10b}$$

Ductile:

$$M_1^0 = R_1 - S/2 \tag{3.11a}$$

$$M_2^{(1)} = R_2 - (S - R_1) \tag{3.11b}$$

Semi-Brittle:

$$M_1^0 = R_1 - S/2 \tag{3.12a}$$

$$M_2^{(1)} = R_2 - (S - \eta R_1) \tag{3.12b}$$

The first set of equations, Eqs. 3.10a-b, shows the safety margins just discussed, namely, those for a structure composed of brittle members. The second set of safety margins, Eq. 3.11a-b, shows the safety margins for a perfectly ductile system. The difference between the assumed element or member post-failure behavior is shown in Fig. 3.5. For the ductile case, M_1^{first}, associated with the intact structure, is unchanged from that for the brittle case. $M_2^{(1)}$, however, now reflects the fact that after failure of Member 1, it retains its force at level R_1 under subsequent deformation of the system. This retained force is, in effect, a reduction in the load that R_2 must carry, vis-a-vis that in the brittle case. Compare Eq. 3.10b with 3.11b. More generally, we can consider the semi-brittle behavior shown in Fig. 3.5 by introducing a factor η, which describes the fraction of the original capacity, R, which remains post-failure in the member. The fraction η ranges from zero to one for the brittle through ductile behavior. For this range of possible member behaviors, the safety margins are shown in Eqs. 3.12a-b. Again, M_1^{first} is unchanged, and $M_2^{(1)}$ appears as the capacity of Member 2 minus the applied load, reduced by the residual capacity in Member 1. In all three cases, the intact structure safety margins are the same, and in fact, are those associated with a first-member failure definition of system failure, as discussed at the end of the preceding discussion of statically determinate or series systems. The particular representation of member behavior shown in Fig. 3.5 is common in current structural system reliability analysis because of the simplicity it introduces. It was used, as well, in our case study, which will be discussed in the Chapter 4. This representation satisfies the condition that the member states be binary (intact and failed), and it permits the replacement of a failed member by its post-failure force (provided there is no subsequent unloading of the failed member).

Union of Failure Sequences. The calculation of the probability of the system failure probability is facilitated by the fact that the failure sequences are by construction mutually exclusive events. Therefore, the probability of their union is simply the sum of their individual probabilities:

$$p_{f_{SYS}} = \sum_i P[FS_i] \tag{3.13}$$

The calculation of the probability of each failure sequence is relatively difficult: it requires the calculation of the probability of a set of intersecting events, e.g., Eq. 3.9. If three members were involved, there would have to be another ordering event such as $M_2^{(1)} < M_3^{(1)}$, implying Member 2 will be the second to fail. In addition, there would have to be a third failure event, $M_3^{(2)}$. Under no circumstances will the events in a sequence be independent (if only because every margin appears in two events). Therefore the calculation of the probability of the intersection can never reduce to simply the products of the probabilities of the events. Generally, the different margins will involve common random variables, e.g., the load S. These calculations are therefore non-trivial; only recently have we had general, efficient, if approximate, computational algorithms for this critical step of the total system analysis (*Hohenbichler and Rackwitz, 1983; Madsen et al, 1986*). Let us turn to making general, semi-quantitative insights into these results. For that, we chose to use the new vehicle provided by the formulation in Chapter 2.

System Factor, C_0 R_0, for indeterminate structures. You will recall from Chapter 2 that we have found advantages in formulating the (intact) system failure probability as a sequence of factors, repeated for convenience in Eq. 3.14:

$$p_{f_{SYS}} = p_{DL} \cdot L \cdot O \cdot D_0 \cdot C_0 \cdot R_0 = P[F_1^0] \cdot C_0 \cdot R_0 \tag{3.14}$$

As mentioned above, $p_{DL} \cdot L \cdot O \cdot D_0$ represents, here, simply the probability of failure of the weakest member, more precisely, of the member with the highest probability of failing in the intact structure, i.e., $P[F_1^0]$. The system factors C_0 and R_0 remain to be considered.

The complexity factor, C_0, is unchanged for this structure from that discussed above for statically determinate or series systems. Recall that C_0 may range from one through approximately one plus a sum of normalized member failure probabilities $\sum_{i\geq 2} P[F_i^0]/P[F_1^0]$. Also recall that C_0 will be closer to one if the correlation between safety margins of the members is high (close to one). We turn, then to the redundancy factor, R_0. We choose to study it here in terms of the product $C_0\, R_0$, recognizing that knowing the value of the product, we can find the value of R_0 by dividing the product by C_0, calculated as discussed above. From Eq. 3.14, we realize that the product $C_0\, R_0$ can be written as shown in Eq. 3.15, as the system failure probability divided by the probability of failure of the most-likely-to-fail member:

$$C_0 \cdot R_0 = p_{f_{SYS}} / P[F_1^0] = \sum_i \frac{P[FS_i]}{P[F_1^0]}$$

$$= \sum_i P[FS_i \mid F_1^0] + \sum_i \frac{P[FS_i \mid \bar{F}_1^0]P[\bar{F}_1^0]}{P[F_1^0]} \tag{3.15}$$

in which $\bar{F}_1^0$ denotes the complement of the event F_1^0, i.e., the event that Member 1 is *not* overstressed in the intact structure. The system failure probability has been replaced by the sum of the failure sequence probabilities, Eq. 3.13. Replacing the probability of failure of any sequence FS_i by the product of this conditional probability, given the failure of Member 1, the most-likely-to-fail (MLTF) member, and the probability of that member failing, plus the parallel term associated with Member 1 not failing, we obtain the result in the last line of Eq. 3.15.

Qualitative Prediction of $C_0 \cdot R_0$; Independent Margins. It is useful to study Eq. 3.15, if only qualitatively, in order to gain understanding and insights into what affects the system factor $C_0 . R_0$. First we will consider the simple case in which the margins such as M_1^0 and M_2^0 are probabilistically independent. Afterwards we will consider the impact of strong correlation. The independence assumption is only valid in limited practical cases; based on Eq. 3.6, we conclude that the load variability, V_S, and the resistance correlations $\bar{\rho}_{ij}$ must both be zero. Nonetheless it provides a useful limiting case.

To facilitate our brief, heuristic study further, we shall consider only the more important sequences, namely those that begin with the MLTF member, Member 1, e.g., sequences such as 1, 2, 3..., 1, 3, 2,..., etc. For "unbalanced" structures, i.e., those in which Member 1 is much more likely to be overstressed in the intact structure than any other, these sequences are certain to be the most important contributors to system failure. For more balanced cases, they will still be representative of other sequences*. The advantage of focusing on

*Experience shows that there are, however, some practical problems in which the MLTF-member-initiated sequences are not the predominant or representative sequences.

Member-1-beginning sequences is immediately evident; for all such sequences the terms in the second summation in Eq. 3.15 are zero. If FS_i begins with Member 1, P[$FS_i \mid \bar{F}_1^0$] is zero. Such a sequence cannot begin if Member 1 is not overloaded.

We turn our attention then to the simple conditional probability $P[FS_i \mid F_1^0]$. For clarity, let us take once again the special case of the ideal two-member system (Fig. 3.3b). Our concern is now the particular failure sequence 1-2. Then expanding in terms of the element margins we obtain Eq. 3.16:

$$P[FS_{12} \mid F_1^0] = P[(M_1^0 < M_2^0) \cap (M_1^0 < 0) \cap (M_2^{(1)} < 0) \mid M_1^0 < 0]$$
$$= P[(M_1^0 < M_2^0) \cap (M_2^{(1)} < 0) \mid M_1^0 < 0] \tag{3.16}$$

where the first sequence in Eq. 3.9 has been substituted and conditioned upon the overload of Member 1, i.e., conditioned on the event that the intact safety margin ,of Member 1 is less than zero. This conditioning implies that one event (the second of the original three) can be removed from the intersection associated with failure sequence 1-2, leaving the intersection of only two events: the ordering event, $M_1^0 < M_2^0$, and the damaged-system failure event, $M_2^{(1)} < 0$. The former event, the ordering event, is relatively uninteresting; given that $M_1^0 < 0$, its probability is virtually one. The interesting event is the total failure of the already damaged structure by the subsequent failure of Member 2, i.e., $M_2^{(1)} < 0$. Recall that the form of this safety margin, $M_2^{(1)}$, depends upon the ductility of the members of the system. Several examples of this safety margin, $M_2^{(1)}$, were given in Eqs. 3.10b, 3.11b and 3.12b. In all three cases, the conditioning event, $M_1^0 < 0$, is simply (R_1 - S/2) < 0.

Consider, first, the brittle case when $M_2^{(1)} = R_2$ - S. In appropriate terms, our probability of interest, $P[FS_{12} \mid F_1^0]$ for sequences starting with Member 1 failures becomes simply the conditional probability that the R_2 is less than the load S, given that R_1 is less than half that load. As discussed above we can assume R_1 and R_2 are independent and S is deterministic in the special independent safety margins case we are considering. The value of this probability is one, unless R_2 exceeds two times R_1, which is practically very unlikely unless R_2 has been intentionally designed to be very much larger than R_1 . In the ideal two-member system, for example, members would have been designed for the same value, namely, half of the load, based on their condition in the intact structure. We conclude that any reasonably well balanced two-member design situation, the conditional probability of this sequence, given the first member failure, is very high, perhaps, close to one. If there were more members in the system, a similar but less strong statement could be made. The sequences' member-failure events would still involve the strong load-redistribution effects associated with brittle behavior, however. We conclude, therefore, that for perfectly brittle structural systems, the conditional probability of at least one (or a group) of the failure sequences involving an initial failure of the MLTF member (conditioned on that member failing), has a high probability of occurrence, unless the second or some subsequent members are much stronger than one would normally expect in usual member design practice of the intact structure. In short $C_0.R_0$ will be approximately one or more in a realistic, brittle case involving independent margins.

Turning to the ductile case for our 2-member system, and continuing to consider those failure sequences which begin with the MLTF member, we follow a similar set of substitutions. We conclude that the conditional sequence probability of interest reduces to, effectively, the probability that the second resistance R_2 is less than the load minus the first resistance R_1, conditional on the first resistance being less than half the load. This is equivalent to saying that R_2 must be less than some factor, k, times the load, where k is somewhat greater than one half. In fact, we do not expect this number to be much greater than a half because we are, in practical cases, in the fast-falling lower tail of the distribution of R_1. We conclude, therefore, that the conditional failure probability of this sequence, given failure of the MLTF member, is approximately the probability that R_2 is less than about half of the load, or simply, the probability of failure of the second member in the intact structure. But in a reliable structure this is a small probability, and it will therefore contribute relatively little to the summation. If all terms are like this, we would then expect the system factor $C_0.R_0$ to be small, i.e., we would expect the system to be more reliable than its weakest member. This, of course, is what we expect from ductile systems.

In conclusion, following just simple, qualitative arguments, we predict that, if the member margins are independent, brittle systems will have a net-system factor $C_0\ R_0$ of one or more, and perhaps almost equal to C_0 itself. In contrast, ductile systems may possibly have a net system factor $C_0\ R_0$ less than one. Given that C_0 will typically be larger than one for such a system, it implies that R_0, the redundancy factor for ductile systems, may potentially be one to several orders of magnitude less than one.

Effect of High Correlation on $C_0 \cdot R_0$. We expect, of course, that the numerical values of these system coefficients will depend upon several other factors, in particular on the correlation among the various safety margins in the problem. Notice that in indeterminate systems, there may be a multitude of safety margins, not only those associated with failures in the intact system, but also those associated with member failures in partially damaged systems. We shall see some numerical results for C_0 and R_0 in Section 3.4.

First, let us deduce without calculations the results for the important case in which the correlation coefficients among the intact-structure margins go to unity. As we have seen above in Eqs. 3.5, 3.6, and 3.7, the margin correlations can be and, in practice, may often be large. The special case of correlation equal to one is therefore of special interest. Studying Eq. 3.5 through 3.7, we see that if these margin correlations are unity, it implies either that the correlations among the resistance variables are all unity, and/or that the coefficients of variations of all the resistances are zero. In all of these cases, the implication is that the MLTF member's safety margin in the intact structure is with certainty (equal to or) less than that of all the others, i.e., the sequence of failures is certain to start with Member 1. All the terms in the second summation in Eq. 3.15 are zero. Furthermore, since all of the subsequent safety margins, e.g., $M_2^{(1)}$, can also be shown to be perfectly correlated with M_1^0, it implies that we can determine deterministically the sequence in which the members will fail. In different words, when the correlation coefficient among all intact structure member safety margins is unity, there is one, and only one, possible failure sequence. It begins with the MLTF member, Member 1. Utilizing the same arguments mentioned above, we will conclude that if the system is brittle and there is not an unusually large unused capacity in the second or some subsequent member to fail, the probability of this sequence given first member failure is close to one. Because there is now only one term in the summation (Eq. 3.15), we conclude that for the brittle system, the $C_0\,R_0$ term is very close to one. In the discussion in Section 3.1 of series systems, we concluded that C_0 is also one in a system with perfectly correlated margins, implying that R_0, the redundancy factor, is one for such a highly correlated system. In different words, for the perfectly correlated, brittle system, the probability of system failure is virtually equal to the probability of failure of the MLTF member in the system in the intact structure. Both the complexity factor C_0 and the redundancy factor R_0 are unity. In short, the system looks in a probabilistic terms like a single member.

Consider next the ductile system in which the correlations among the safety margins are all unity. Again, there will be one and only one sequence, and it will begin with the MLTF member. Recalling that this case is effectively that in which the member resistances are without randomness (at least with respect to one another*), the reader can convince himself that the conditional sequence failure probability that we are after, $P[FS_1 | F_0^1]$ reduces to the probability that the load exceeds a linear combination of deterministic member capacities, given that the load has exceeded some coefficient times the first member's capacity. The coefficients in the linear combination come from an analysis of the collapse mechanism associated with failure, e.g., Eq. 3.12b for the ideal two-member system. The coefficient in the conditioning event comes from the analysis of the stresses in the MLTF member in the intact structure. In different terms, the probability of this unique sequence, given first-member failure, is the probability that the load exceeds the mechanism's capacity, given that it exceeded the load required to cause first-member yield. This conditional probability, which is equal to C_0R_0, can be expressed as the ratio of two values of the complementary cumulative distribution function (CCDF) of the load: the CCDF of the load evaluated at the system mechanism capacity divided by the CCDF of the load evaluated at the intact first-member capacity. The value of this ratio will be less than or equal to one. The value depends upon the relative value of the (deterministic) strength of the system to the (deterministic) strength of the (intact structure) MLTF and on the coefficient of variation of the load. If that coefficient of variation is smaller, the ratio ($C_0 \cdot R_0$) will be smaller. Hence, we expect the ductile system with perfect correlation to be more reliable than its MLTF first member by this calculated factor $C_0\ R_0$, which can be easily obtained from the CCDF of the load, the deterministic capacity of the "weakest" (first-to-yield) member, and the (known) failure mechanism's deterministic capacity.

If we consider the very special, n-member, ideal parallel structure (with identically distributed resistances), of which our structure in Fig. 3.3b is the 2-member example, then we will find that the conditional probability of the sequence of interest, given first-member failure, is simply the probability that the load exceeds n times the (deterministic) capacity of any one member, given that the load divided by n exceeds the ("deterministic") capacity of one

*That will hold in the $\rho_{ij} = 1$ (for all i,j) case, in which case all resistances depend functionally on R_1, and we can "shift" this resistance's variability to the load for the purposes of this discussion, leaving us with the simple case of a random load and deterministic capacities.

member. Those two events are, of course, the same, implying the conditional probability is unity. This tells us in different terms that in this structure, the force deformation diagram of the structure, as a whole, is also elasto-plastic, i.e., the ultimate capacity of the system is is reached when the capacity of a member is reached. The system factor C_0 R_0 is unity. But we know from our discussion of series systems that for perfectly dependent margins C_0 is also unity, implying that R_0 is unity. The interesting conclusion is that, although the system is ductile, because it is not only perfectly balanced, i.e., all members are equally stressed under the intact load condition, but also because the member capacities are effectively deterministic, (i.e., there is no relative variability) there is no effective redundancy in the system; the system failure probability coincides with that of the MLTF member. Again, in the ductile case, the well-balanced system behaves like a simple member.

Conclusions. When margins are independent, $C_0 \cdot R_0$ for ductile systems may be very low (very good) even for ideal (well balanced) systems. For well balanced, independent brittle systems $C_0 \cdot R_0$ may exceed one. When the correlations among member safety margins in the intact structure are unity, the behavior of the ideal system reduces to very nearly that of the MLTF member. Both systems complexity and system redundancy are effectively unity. This conclusion is true for both brittle and ductile systems. This high correlation, recall, accompanies high load-to-resistance variability, or high correlation among resistances, or some combination of the two. The first condition, if not the second, is perhaps a likely one in many offshore structure applications.

As a general conclusion, we find that, given realistic moderate to high margin correlations, if the system is what we shall call "well balanced," i.e., if there are many members that are highly stressed in the intact structure, particularly those that will be called upon to carry extra member forces subsequent to the failure and unloading of a brittle first-member failure, or loads in addition to that causing failure of a ductile-first member failure, we should not expect significant system effects. C_0 and R_0 will be close to one. System failure can be expected to follow shortly upon first-member failure. The analysis of first-member failure is, of course, very simple. In this case, it is simply the failure probability of the MLTF member. The probability of failure of this member can be controlled simply by a probability-based, member-level code of the type in effect in Norway and under current consideration by the API . The conclusions here imply that there need be no system factor (benefit or disbenefit) term for the code *if* correlations are high and the system is well balanced.

The key word in the discussions above may well be "well balanced." The balance is relative to the load under consideration. There are many design situations in which this balance does not occur, even if it is the designer's objective. One interesting case is when other loading conditions, such as waves from a different direction, construction or launch-and-tow loads cause certain members to be lightly stressed under any particular set of waves loadings upon the intact structure. These members may provide the lack of balance, and hence the backup capacity, that will create system redundancy, whether the system is brittle or ductile.

Also an exogenously damaged member can create an out-of-balance system. In this case, as we discussed in Chapter 2, we may expect that due to the possible localization of stress increases, the MLTF member, i.e., the most heavily stressed member, may be significantly more highly stressed than many of the other members in the structure. This situation implies lack of balance, and again, the possibility of strong system effects, as measured by C_0 and R_0, or in the notation of Chapter 2, $\bar{C}_j$ and $\bar{R}_j$.

A final comment about the conclusions above. They are all based on the application of a single static load to the structural system. The analysis treats only the ultimate capacity of the system in load terms; it does not distinguish between two systems whose system-level force deformation diagrams are different, i.e., brittle vs. ductile. The analysis treats only the absolute capacity of the system in load terms. We know, however, that the ductile system is preferred to the brittle system for many reasons. In particular, the reason is that the true loadings on the structural system will not be of the (as modeled) static, pushover type. Rather, we would expect repeated load reversals when waves attack shallower, static structures, or stress reversals when dynamic wave behavior or seismic behavior is the governing load condition. Under these repeated load conditions, the ductility of the system will be critical to its survival. More complex analyses are, of course, necessary to reflect these reversals.

3.3 Numerical Results: C_0 and R_0 of Ideal Parallel Systems

To quantify the general trends anticipated in Section 3.2, we present here some numerical results for ideal parallel structural systems, by which we mean systems like that in Fig. 3.3b, but with an arbitrary number of members, n. The model is characterized by common member stiffnesses, and by all member capacities having the same means and coefficients of variation. It is of course a perfectly well-balanced system. This problem has been studied by many investigators (e.g., *Daniels, 1945; Stahl and Geyer, 1985; Hohenbichler and Rackwitz, 1983;* etc.).

It should be pointed out that deterministically this ideal system would appear to have *no* complexity or redundancy. All members would fail simultaneously. Any complexity effect ($C_0>1$) or any redundancy effect ($R_0<1$) that we find here might therefore be called *probabilistically induced*. They come about because, given random resistances, one member will be the weakest and fail first, probably at a capacity less than the mean (hence complexity); others, however, may be adequate to "pick up any slack" and produce a higher system capacity (redundancy). A further implication is that these particular complexity and redundancy effects are not obtainable by deterministic analysis; they represent results that can be the product only of a systems reliability study.

Guenard's Results. A convenient set of results was obtained by Guenard *(1984)*, where the results are presented in his Figs. 2.12 through 2.15, which we repeat here for convenience as Figs. 3.6 through 3.9. These figures show, through the so-called safety index β, the failure probability of these systems as a function of the number of members, the ductility/brittleness (η in Fig. 3.5), and various coefficients, such as the member capacity coefficient of variation, the correlation coefficient among the capacities, and the safety factor (defined as the ratio of the mean resistance of the single-member structure to the mean applied load). For equity in the comparison, as the number of members is increased, their common mean resistance value is reduced proportionally to insure that the system is deterministically, or "in the mean," equivalent from case to case. (The probability distributions are joint lognormal.) The reader is invited to study these figures and draw his own conclusions about the effects of correlation, brittleness, coefficient of variation, etc. It is useful to remember that a β of 3 corresponds to a failure probability of approximately 10^{-3}, and that the probability of failure reduces approximately two orders of magnitude per unit of β in the range $\beta=3$ to $\beta=5$ or 6, and about three orders of magnitude with unit increase of β from 5 or 6 through 8 or 9.

Guenard's example is for a special case, namely an ideal system with deterministic loading and random resistances. We can, however, interpret the results more broadly. It can be shown that, with some degree of approximation, we may let his coefficients of variation on resistance (denoted V) be interpreted as the coefficient of variation of the safety factor, i.e., the random resistance divided by the random load effect in an individual member in the intact structure. This V value is approximately $\sqrt{V_R^2 + V_S^2}$. (This V is also approximately the standard deviation of the (log) safety margin.) Also, we may interpret his correlation coefficient between resistances (denoted ρ) as approximately that between the (log) safety margins of the members in the intact structure. From Eqs. 3.6 and 3.7, we therefore can make the following interpretations. When the correlation coefficient in his example is zero, it implies that both the load coefficient of variation is zero and the correlation coefficient between the resistances is zero. When the correlation coefficient is 0.5 in his analysis, it implies that, if the resistance correlation coefficient, $\bar{\rho}_{ij}$, is 0, then the load coefficient of variation equals the resistance coefficient of variation, or, alternatively, that if the load coefficient of variation is 0, then the correlation coefficient between resistances, $\bar{\rho}_{ij}$, is 0.5. (This broader interpretation is strictly valid only in the intact structure. The approximation is apparently reasonably accurate, however, for the complete system failure problem.) In short the figures represent a relatively broad suite of cases, if we interpret the coefficients properly. For some cases of practical interest, however, the coefficient of variation and correlation coefficients here may be relatively low. Therefore, additional results are provided below to supplement the "re-interpreted" Guenard results.

Next, in order to use these results for the purpose of looking at C_0, R_0, and C_0R_0, we realize that in our terminology the probability of the failure of the MLTF member, which we denote $P[F_1]$ or $p_{DL} \cdot L \cdot O$, is simply the probability associated with the β of the system at $n=1$ in his figures. This is the probability of failure of a single element in the intact structure no matter how many members the system has, $n=1,2, \cdots$ or 8. For example, in Figs. 3.6 and 3.7 this β value is slightly less than 4 implying a first member failure probability of approximately 10^{-4}. Finally, Guenard's curves give the system failure probability (or rather β). We can obtain the net system factor $C_0 \cdot R_0$ by simply dividing the system failure probability by the first member failure probability. For example, in Fig. 3.7 the upper curve for $n=4$ to 8 shows a system β of approximately 4.5 to 5, implying a system failure probability of about 10^{-6}. Therefore, the system factor $C_0 \cdot R_0$ is approximately $10^{-6}/10^{-4}$, or 10^{-2}.

Next, we consider the complexity factor C_0. For the low values of correlation coefficient considered in Guenard's figures and for the relatively low values of n here, we can assume that C_0 is approximately equal to n, the number of members, for the reasons discussed in Section 3.1. The value of C_0 will be approximately independent of ρ, the coefficient of variation, and the system safety factor (i.e., ratio of mean resistance to mean intact load effect) for the range of parameters considered in Guenard's figures. It is, of course always independent of η, the post-failure member ductility measure, because the complexity factor is independent of post-failure behavior; it is only first-member failure related.

Knowing now both C_0 and the product $C_0 \cdot R_0$, we can obtain R_0 by simple division. Plots of approximate values of C_0, R_0, and the product $C_0 \cdot R_0$ as a function of the number of members in parallel in the ideal system (for several different parameter values and ductility levels) are shown in Figs. 3.10 and 3.11. The curves include cases for ρ=0 and ρ=0.5. Recall from the discussions in Section 3.2 that if ρ=1 for this perfectly balanced system, we know that C_0 and R_0 and $C_0 \cdot R_0$ will all be unity. Therefore, we have a relatively broad spectrum of member (log) margin correlation coefficient values represented (0, 0.5, and 1). Assuming that the correlation between member resistances $\bar{\rho}_{ij}$ is zero, this spectrum of member margin correlation coefficient values represents (Eq. 3.7) a ratio of coefficient of variation of the load to that of the resistance ranging from zero to unity to infinity.

The reader is again invited to study for himself the effects of correlation, uncertainty (COV), and ductility upon system complexity and system redundancy factors. The product $C_0 \cdot R_0$ is also plotted in these figures even though it represents simply a re-scaling of Guenard's figures. Casual inspection reveals that for high ductility the redundancy factor can be very small (i.e., very good), but only for small correlation coefficients and lower coefficients of variation. As both of these factors grow and as the brittleness increases (i.e., as η decreases), the redundancy factor becomes much larger, i.e., there is less effective redundancy in the system.

Additional Results. A second set of ideal-system results produced for this project using Guenard's program is shown in Figs. 3.12, 3.13, and 3.14. They are designed to show the results in a more useful parametrization, i.e, for a fixed MLTF member probability, $P[F_1]$. The mean resistances are selected so that for each structure $P[F_1]$ is fixed at 10^{-3} or 10^{-5}. We work here directly with $M_i^*=\ln[R_i/(S/n)]=\ln(R_i)-\ln(S)+\ln(n)$. R_i and S are assumed to be jointly log-normally distributed with standard deviations of logs (or approximately COV's) of σ_R^* and σ_S^*; the log capacities are assumed to be jointly normal with common correlation coefficient $\bar{\rho}_{ij}$. (This is also approximately equal to the correlation coefficient of the capacities themselves.) The load is independent of the resistances. Eq. 3.6 holds for ρ_{ij}^*. The results shown in Fig. 3.12, 3.13, and 3.14 are therefore valid for all cases in which the indicated ρ_{ij}^* and $\sigma_{M^*}=\sqrt{\sigma_R^{*2}+\sigma_S^{*2}}$ hold. For example, if $\bar{\rho}_{ij}=0$, then ρ_{ij}^* is approximately the ratio of the (squared) load COV to the sum of the (squared) load and resistance COV's; and σ_{M^*} is the square root of that sum, i.e., approximately the COV of the antilog of M (i.e., the COV of the safety factor, $R_i/(S/n)$). The results are shown for a range of practical values. We see that if ρ_{ij}^* is as large as 0.9, as Chap. 4 suggests it might well be in practice, then neither C_0 nor R_0 differs as radically away from unity as we have seen in previous examples. For $\rho_{M_iM_j}=0.9$, the redundancy factor R_0 can at best just offset the complexity factor, which is only 2 to 4 at $n=8$ (for 10^{-3} and 10^{-5} respectively) at this high correlation level. It requires ductility to do even this well; if any significant brittleness exists, the complexity "wins" and the net system factor follows C_0. In this range the general trends are mildly sensitive to absolute probability level. C_0 is somewhat larger and R_0 is somewhat smaller for more reliable systems. Lower margin correlation (e.g., 0.5) improves R_0 only in the ductile case, permitting significant net system redundancy ($C_0 \cdot R_0<1$). Given any significant brittleness, however, C_0 again drives the net system factor, and to values close to n (e.g., in excess of 6 for n=8).

Although this is a very idealized system, it can be an extremely informative one for purposes of obtaining an understanding of the impacts of randomness, correlation, and brittleness on system redundancy. It, in fact, may be rather accurately representative of well-balanced indeterminate structural systems. As we have discussed, this ideal model will be characteristic of redundant structures subjected primarily to single dominant loading situation in which the member sizes have been selected very nearly in proportion to the stress distribution in the intact structure. In the case study of a large jacket platform (Chapter 4), we found that the simple ideal model here could represent rather well the behavior of that more realistic system provided proper interpretation was made. Most of the critical failures of members in that structure took place at one level of panels of vertical X bracing. (The same effect was observed in a companion structure designed with K rather than X bracing.) In the four vertical bents of the eight-legged structure, each such panel of the X (or K) type can be represented as an "equivalent member" of the ideal system here. Because there are four bents, there are four "members" in this ideal system. The coefficients of variation, the correlation coefficients, and brittleness factors for the equivalent members have to be estimated from those of the two-member "system" making up the X (or K) braced panels. How this can be done is discussed in detail in a companion report *(Nordal et al, 1987)*. Needless to say, the X-braced "member" will have a larger "equivalent ductility measure", η, than the K-braced. With the assumptions in that case study, the equivalent correlation coefficient among the (log) safety margins of these four "members" was very large, 0.96. We can anticipate, then, even from our simple arguments here that the systems will behave as if their complexity and redundancy factors were close to 1. These conclusions hold for the analysis of that particular hypothetical jacket in its intact (undamaged) condition and subjected to a single extreme wave load from one perpendicular direction. There is ample evidence that this case study structure is atypical of many real offshore structures, because of the particular way this hypothetical case was designed originally. The result of that design process left all vertical bracing systems almost uniformly highly stressed, providing no backup capacity should one of the bracing panels fail, thereby eliminating that potential source of (post-first failure) residual strength or what we might call here "deterministic redundancy", i.e., redundancy provided by "lack of balance" of the member capacities with respect to this particular loading case. Also because the effective correlations among safety margins are so high, the potential for what we have called probabilistically-induced redundancy is also lacking. Recall that "probabilistically-induced redundancy" (R_C<1) is apparent in Fig. 3.11 in (ideally balanced) systems only for those cases with both low correlation coefficients and high ductility.

3.4 The Failure Path Formulation

We turn to one final point of discussion of the formulation of system reliability problems. This modified formulation was in fact used by Guenard to obtain the results which we have just discussed. Recall Eq. 3.9, which presented the expansion of the event {system failure} into safety margin events. We find there the union of two failure sequences and within each failure sequence the intersection of three events. Consider the first sequence. The first of the three events we identify as an ordering event; the second the event that the first member failed in the intact structure; and the third event, the event that the second member failed in the partially damaged structure. It is observed that the first or *ordering event* $(M_1^0 < M_2^0)$, is of an entirely different nature than the latter two. The last two events are member-failure related. The first-member failure event will always be a small probability event; the second may or may not be, depending on many factors as we have seen. The ordering event, however, will always have a relatively large probability. In a well-balanced, two-member system the probability that the first member margin is less than the second member margin (in the intact structure) is approximately a half. For the perfectly ideal parallel system, it is exactly equal to $1/n$; in our simple example here $n=2$.

It has been suggested by Bjerager *(1984)* and others that a useful (and conservative) approximation in this event formulation of the failure sequences will be to eliminate the ordering event from the failure sequence. Because we are eliminating an event in an intersection, we are, of course, obtaining an event which is "larger" than (i.e., contains) the original event and which will, therefore, have a probability equal to and greater than the original event. This approximate representation of the system failure is shown in Eq. 3.17 for the ideal parallel structure (see, in contrast, Eq. 3.9):

$$F_{system} \approx [(M_1^0 < 0) \cap (M_2^{(1)} < 0)] \cup [(M_2^0 < 0) \cap (M_1^{(2)} < 0)] \qquad (3.17)$$

At Stanford, we have adopted the habit of calling this formulation of the problem a *failure path formulation* as opposed to the more precise *failure sequence formulation* discussed above. Notice that in a failure path representation of a two-member structure, we still have the union of two events, each of which is associated with a particular order of member failures, the association arising through the sequence of intact and partially damaged structures that must be analyzed. Within each of these failure paths we continue to have an intersection of events. Now, however, these are limited only to member failures events and they do not contain explicitly the ordering events that exist in the system failure sequence representation.

It is believed, although it has not been formally proven, that the approximation is not seriously in error. It is always conservative, and in some circumstances ("monotone structures", see *Bennett, 1986*, and *Karamchandani, 1987)* it is exact. The reasons for making this approximation are primarily convenience and computational efficiency. It is preferable computationally to work with the intersection of events with small probabilities. The approximations available for carrying out such calculations in problems involving large numbers of random variables are accurate only for this case. We do not have good available computational schemes for the circumstance in which there is included in the set of intersections an event with relatively high probability, such as the ordering event. The implication is that the calculational inaccuracy in computing the more accurate formulation of the problem using the failure sequence rather than the failure path may overcome any weakness in the event representation associated with the simpler and more accurately calculated failure path.

An implication of the failure path formulation (see Eq. 3.8):

$$F_{system} = \bigcup_i FS_i = \cup_i \cap_j F_{i,j}^{ordered} \approx (F_1 \cap F_2^{(1)}) \cup (F_2 \cap F_1^{(2)})$$

$$= \bigcup_i \bigcap_j F_{i,j} = \cup_i FP_i \qquad (3.18)$$

is that the failure path events, denoted FP_i (as distinct from the failure sequence events denoted FS_i), are not mutually exclusive. Therefore, when calculating the probability of failure of the system, we cannot use the simple summation of probabilities indicated in Eq. 3.13, but we must account, in principle at least, for the fact that the failure path events are not mutually exclusive and allow for their intersections. In the final lines of Eqs. 3.8 and 3.18, we indicate that both the failure sequence formulation and failure path formulation of the system failure event can be represented as a union of failure sequences or failure paths, which in turn can be represented as an intersection of member failure events denoted $F_{i,j}^{ordered}$ and $F_{i,j}$. In the former case, we emphasize that the sequence of these member failure events must be maintained strictly through ordering events such as those shown in Eq. 3.9.

3.5 Summary

In this chapter, we have shown how to formulate in terms of events and random variables the system reliability analysis problem for determinate (series) and indeterminate (parallel) load-sharing structures. In all cases, we have reduced the system failure event down to a statement involving unions of events. In the parallel systems, these involve the unions of events which are in turn intersections of events. Those events are all member-failure events that can be expressed in terms of member safety margins, which in turn can be written in terms of capacities and load effects. In the simple, brittle, semi-brittle and ductile systems we have discussed in detail and as illustrations here, these member failure events can be written in terms of simple linear combinations of member capacities, R_i, and applied load, S.

In practice, these capacity and load random variables may themselves in turn be functions of several so-called *basic* random variables which all together make up a vector $\underline{X}$. These may include material properties, dimensions, wave heights, periods, etc. In fact, current practice is to write, instead of margin-based failure events, the more general statement: $g(\underline{X})<0$. The literature is replete with methods and techniques for actually carrying out the computations of probabilities of unions of intersections, each involving an event described by the negativity of a function g of a vector of random variables $\underline{X}$. It is not our intention in this report to discuss these computational methods except as they impact the questions of system reliability formulation. One such example was discussed just above where the ineffectiveness of current computational capabilities with respect to intersections of events in which some of those events are of relatively large probability while others are of relatively small probability was cited as a reason for replacing the failure sequence formulation by the failure event formulation.

The structural system reliability analysis formulation that we have presented in this chapter is capable of handling a rather broad set of structural systems. But the representation of the mechanical behavior of these elements is obviously extremely limited; the simple two-state member behavior may be inadequate for proper representation of structural system behavior, particularly in the representation of post-member-failure such as post-buckling behavior. The limitation to a single, monotonic, static load system is also a limitation on this method. Questions of limitations of the mechanical formulations, the probabilistic formulations and the computational procedures will be discussed in Chapter 6 to follow. We will find that it is possible to enhance the mechanical assumptions made in this chapter without major reduction in computational efficiency for certain classes of problems. For mechanically more complex problems, for example those involving structural dynamic behavior, more general systems reliability formulations and computational methods are necessary. Although such techniques exist they are currently too inefficient for practical application to large structures. One of the objectives of all current research in systems reliability is to expand the capabilities and efficiency of these techniques for more general applicability.

The current chapter has, however, indicated how a relatively realistic set of structural member behavior assumptions can be represented in a structural systems reliability formulation. We have used that formulation to show how factors such as the system complexity factor and the system redundancy factor depend upon post-failure mechanical behavior and upon probabilistic parameter values such as coefficients of variation and correlation coefficients among important random variables; namely, resistances and loadings. We believe that a careful study of these relationships will bring engineers new insights into the reliability and safety of large structural systems, insights that are not available in deterministic systems analysis nor in member-level structural reliability analysis. We hope, therefore, that this chapter has helped demonstrate some of the potential advantages of structural systems reliability.

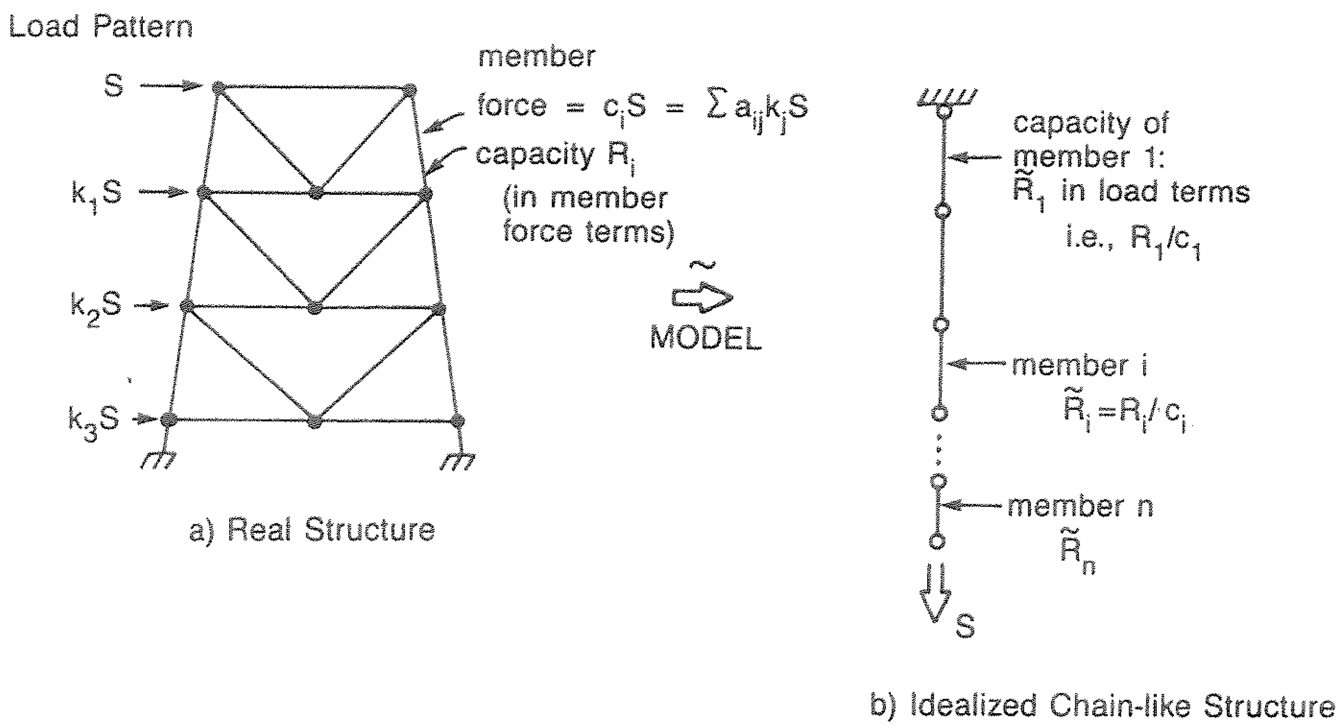

Fig. 3.1: Statically Determinate Structure.

(Notation: upper case letters are random variables, e.g., R_i; lower case letters are specified constants, e.g., k_i).

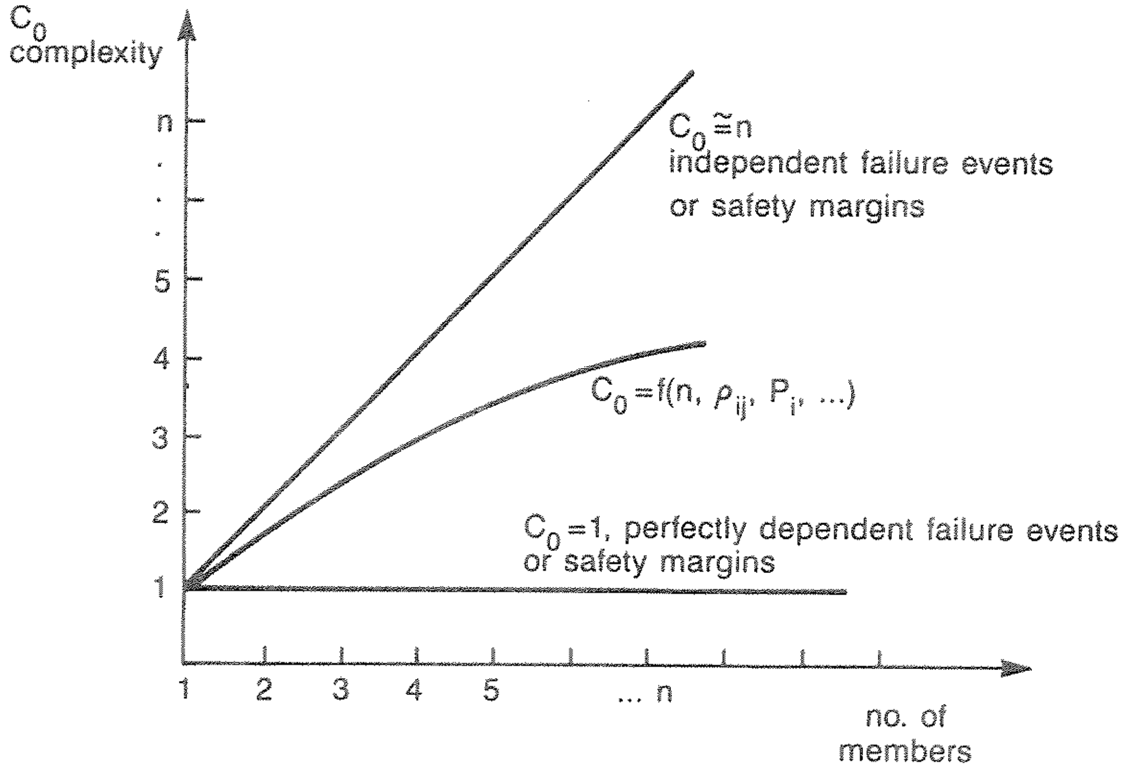

Fig. 3.2: Complexity factor, C_o, for the ideal system in which all members have the same (marginal) failure probability, $P[F_i]$. For the general problem C_o ranges from 1 to approximately $1+\sum_{i\geq 2} P[F_i]/P[F_1]$.

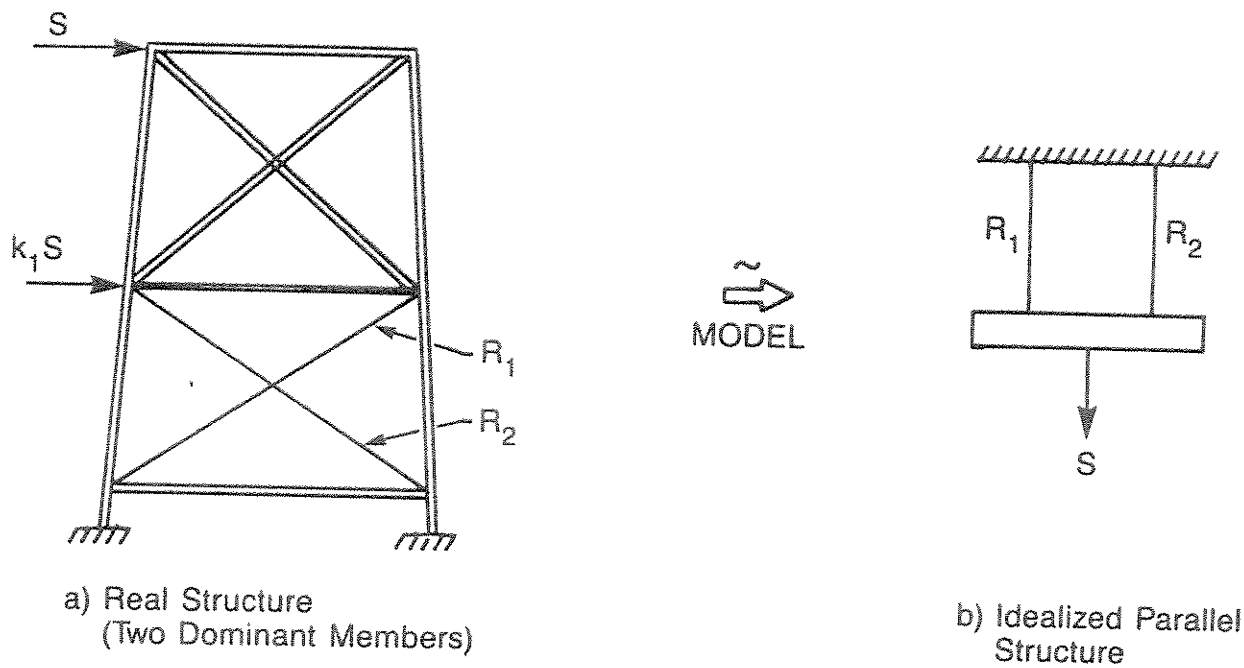

Fig. 3.3: Redundant Structure. (See Fig. 3.1 for definitions and notation.)

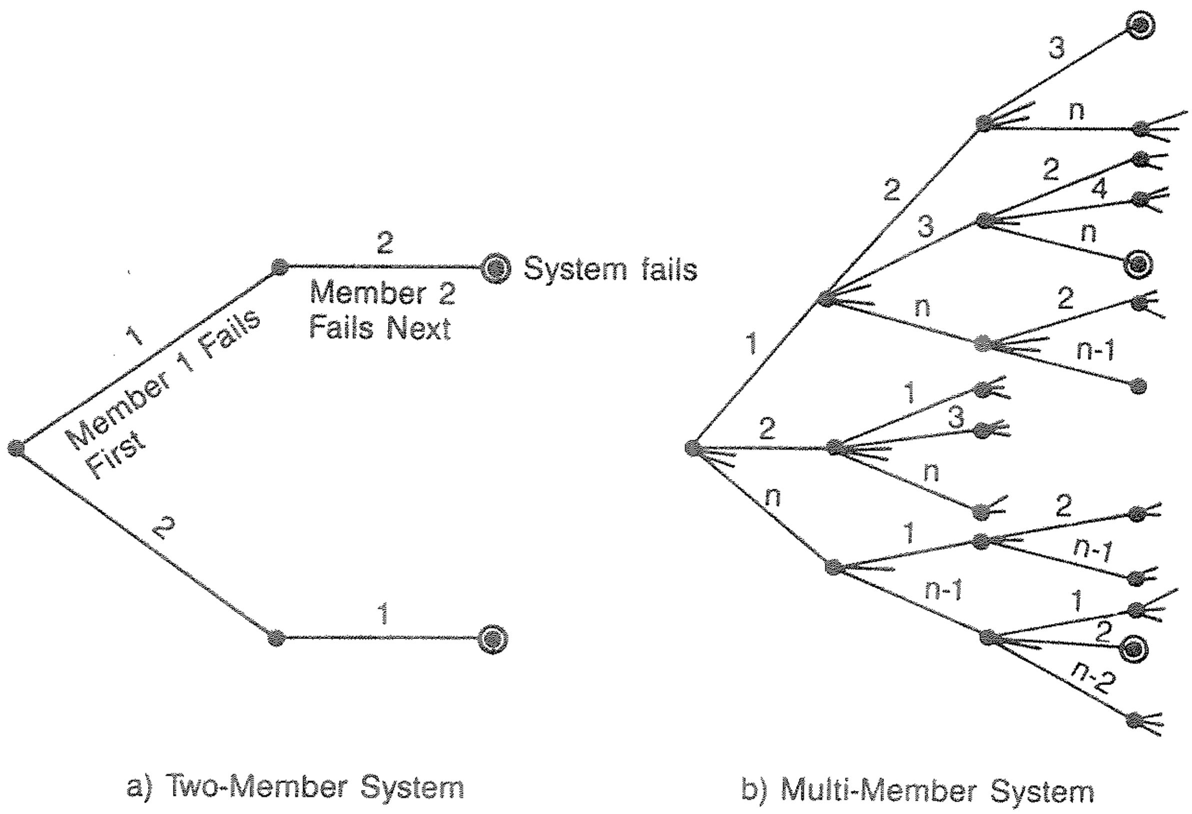

Fig. 3.4: Failure-Sequence Trees.

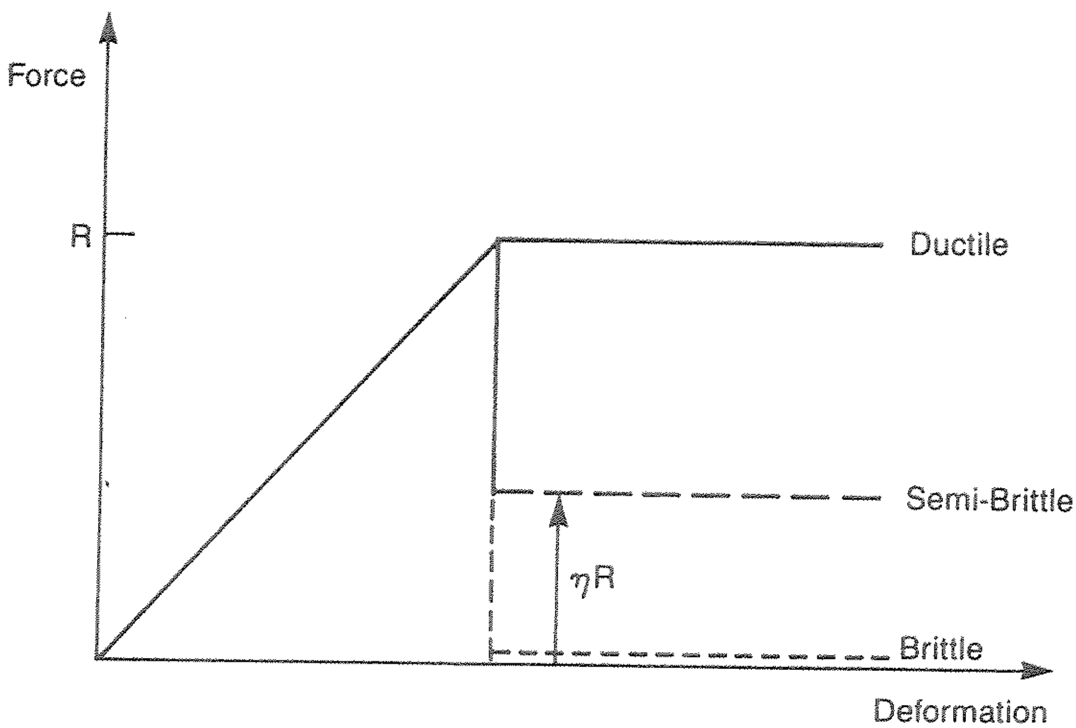

Fig. 3.5: Several Simple Alternate Member Force-Deformation Assumptions.

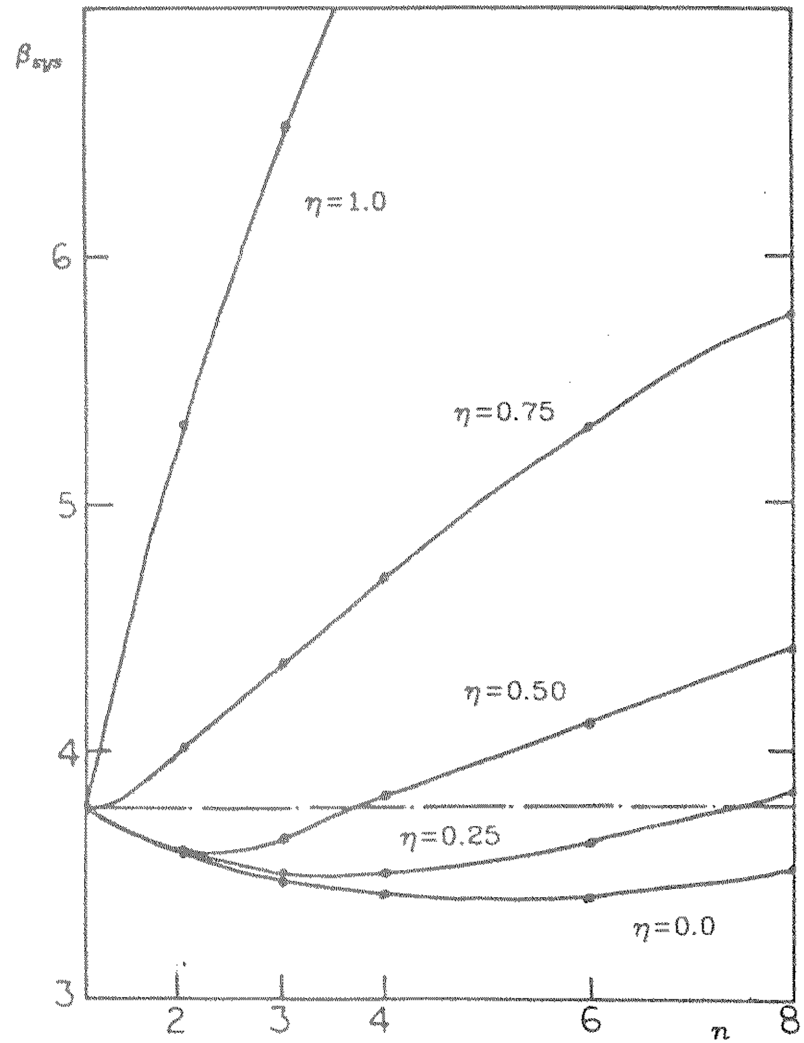

Fig. 3.6: Results of Ideal Parallel System Analysis.
Coefficient of variation of safety factor (V)=0.1.
Mean safety factor $(\overline{SF})$=1.47.
(Log) margin correlation (ρ)=0.

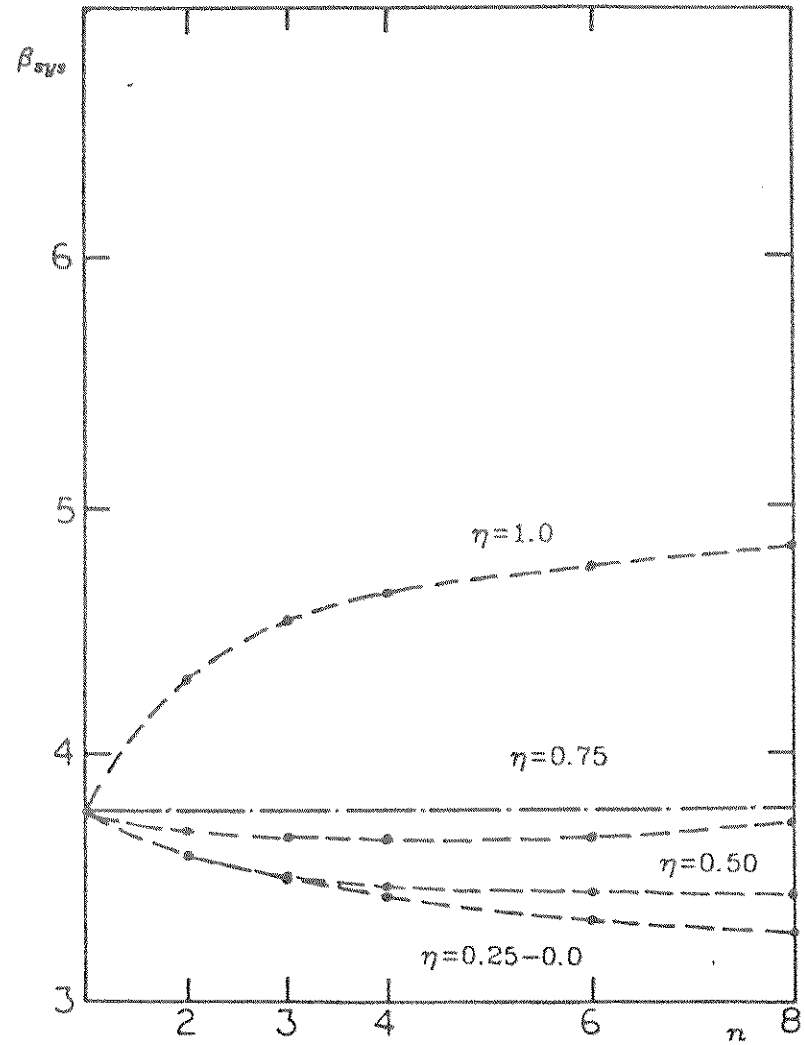

Fig. 3.7: Results of Ideal Parallel System Analysis.
Coefficient of variation of safety factor (V)=0.1.
Mean safety factor $(\overline{SF})$=1.47.
(Log) margin correlation (ρ)=0.5.

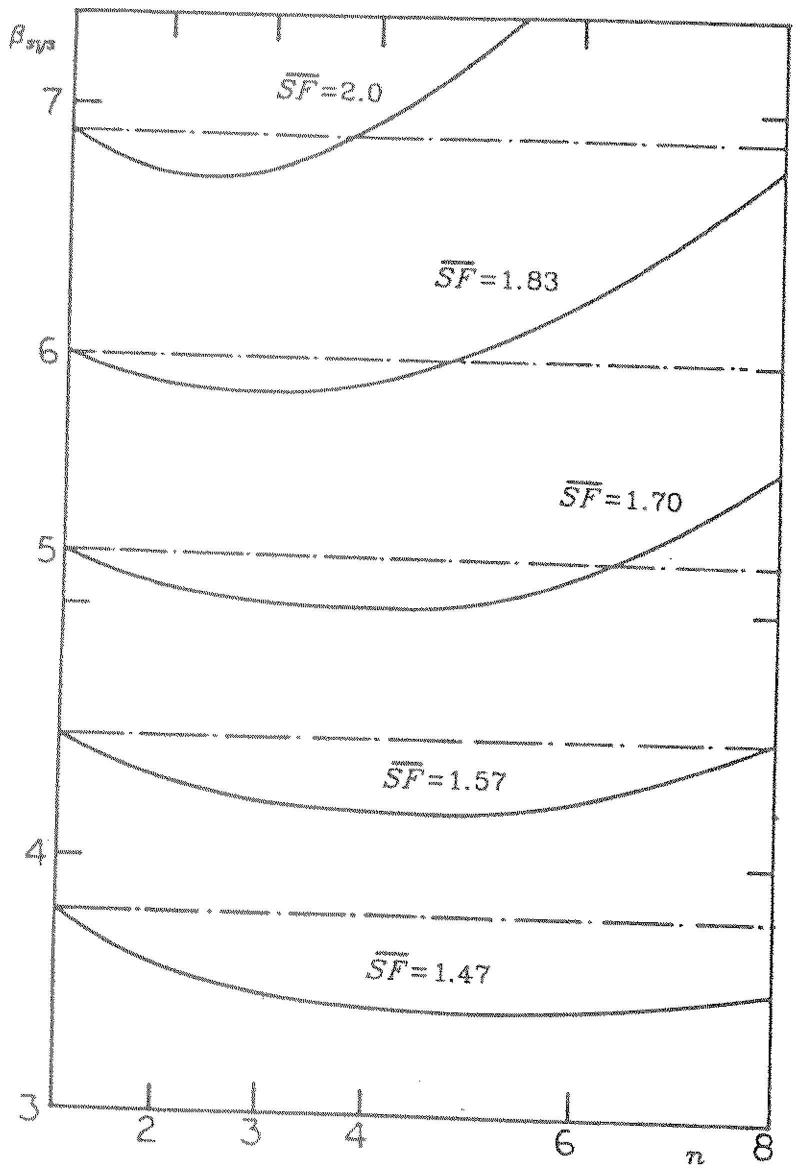

Fig. 3.8: Influence of $\overline{SF}$ on the Redundancy of the Brittle Parallel System (V=0.1).

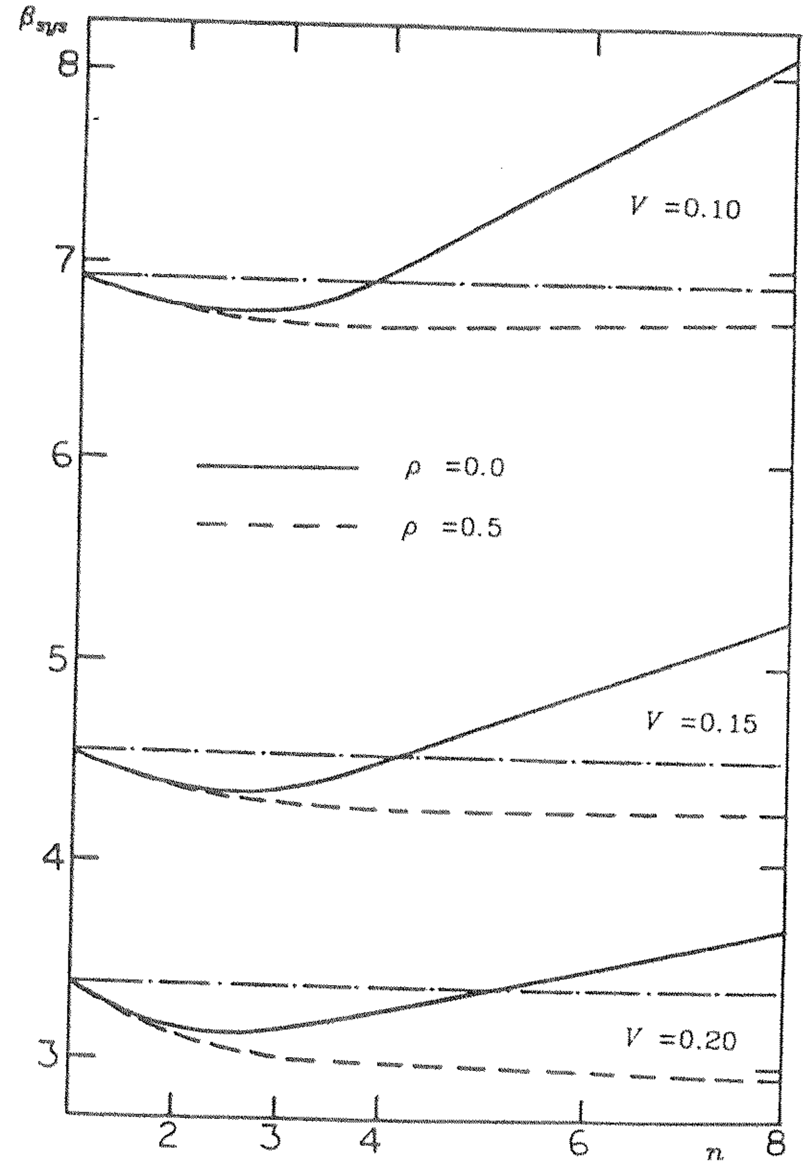

Fig. 3.9: Influence of V on the Redundancy of the Brittle Parallel System ($\overline{SF}$=0.1).

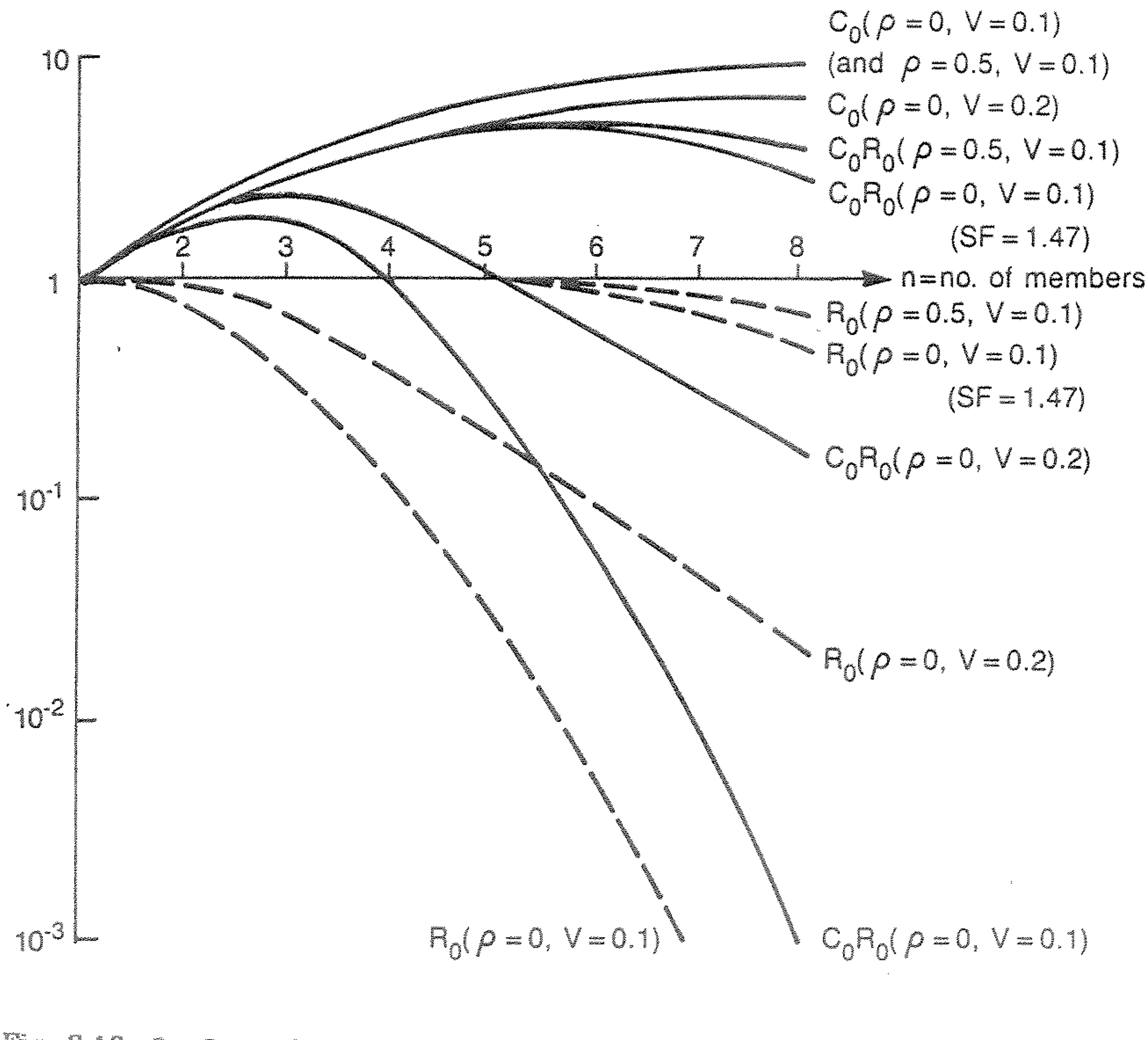

Fig. 3.10: C_o, R_o, and C_oR_o — **Complexity, Redundancy, and Net System Factor for Ideal Parallel Systems of the Perfectly Brittle Type** ($\eta=0$). $COV(M_i^o)=0.1$ or 0.2 as shown. $\rho(M_i^o, M_j^o)=0$ or 0.5 as shown. Note: $M_i^o=R_i-S/n$. $SF=2.0$ except as shown. (C_o ($\rho=0$, $V=0.1$) curve applies to both $SF=1.47$ and $SF=2.0$.)

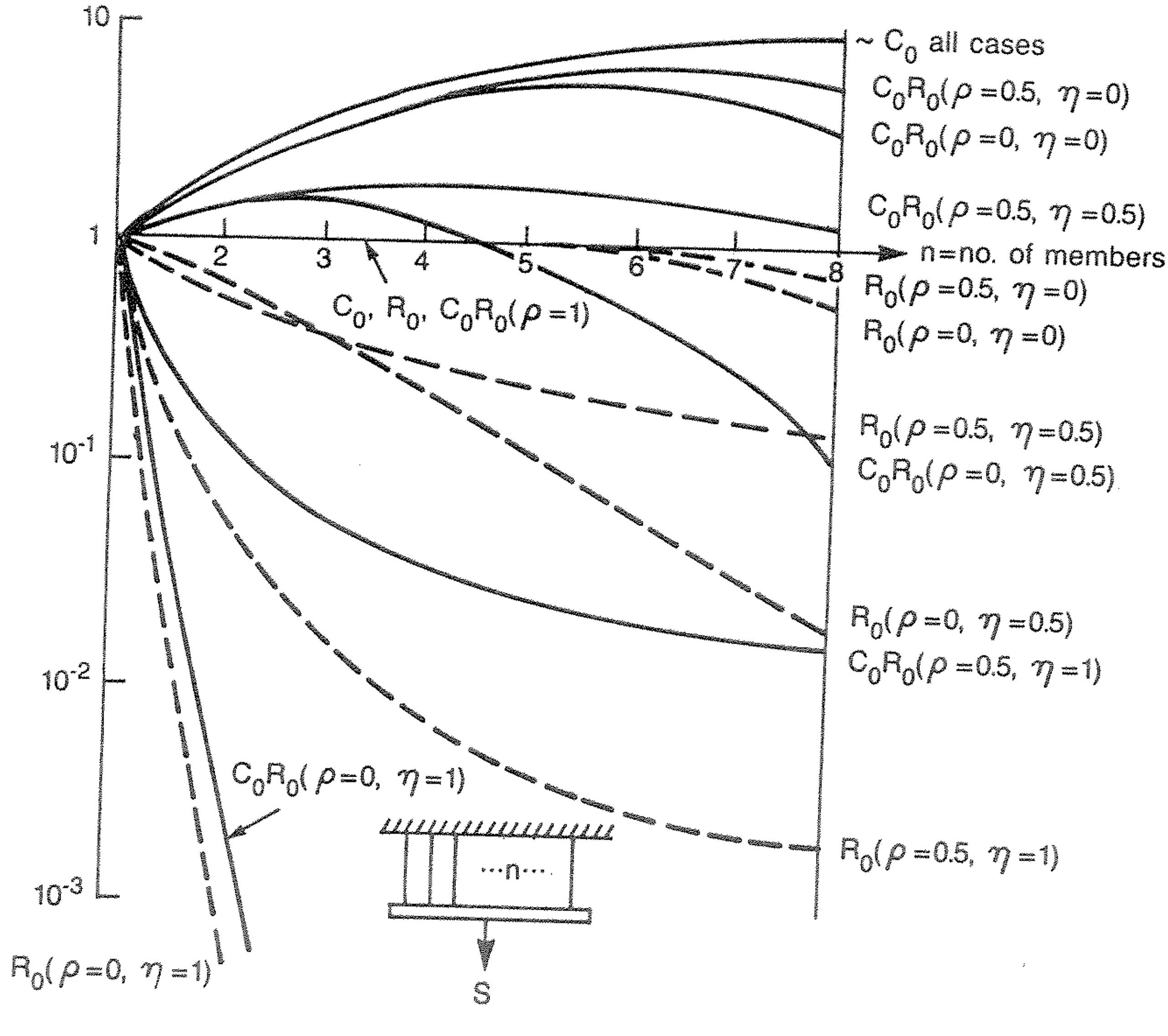

Fig. 3.11: C_o, R_o, and C_oR_o — **Complexity, Redundancy, and Net System Factor for Ideal Parallel Systems.** $COV(M_i^o)=0.1$; $\rho(M_i^o, M_j^o)$ as shown, ductility η as shown. Note: $M_i^o=R_i-S/n$.

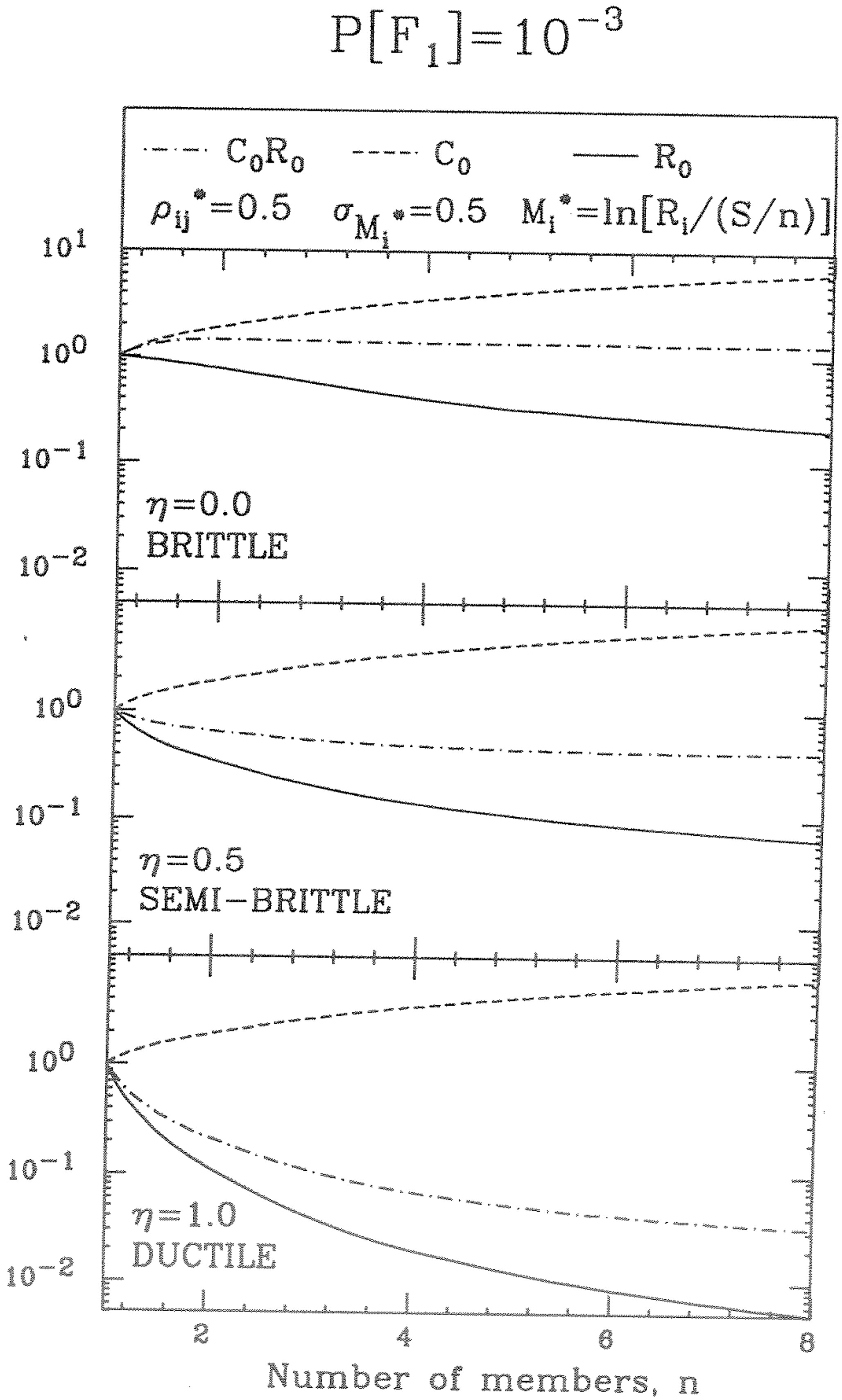

Fig. 3.12: C_0, R_0, and C_0R_0 — **Complexity, Redundancy, and Net System Factor for Ideal Parallel Systems**. Lognormally-distributed resistances and load with parameters shown. Mean resistances specified to make $P[F_1]$ the same for all structures.

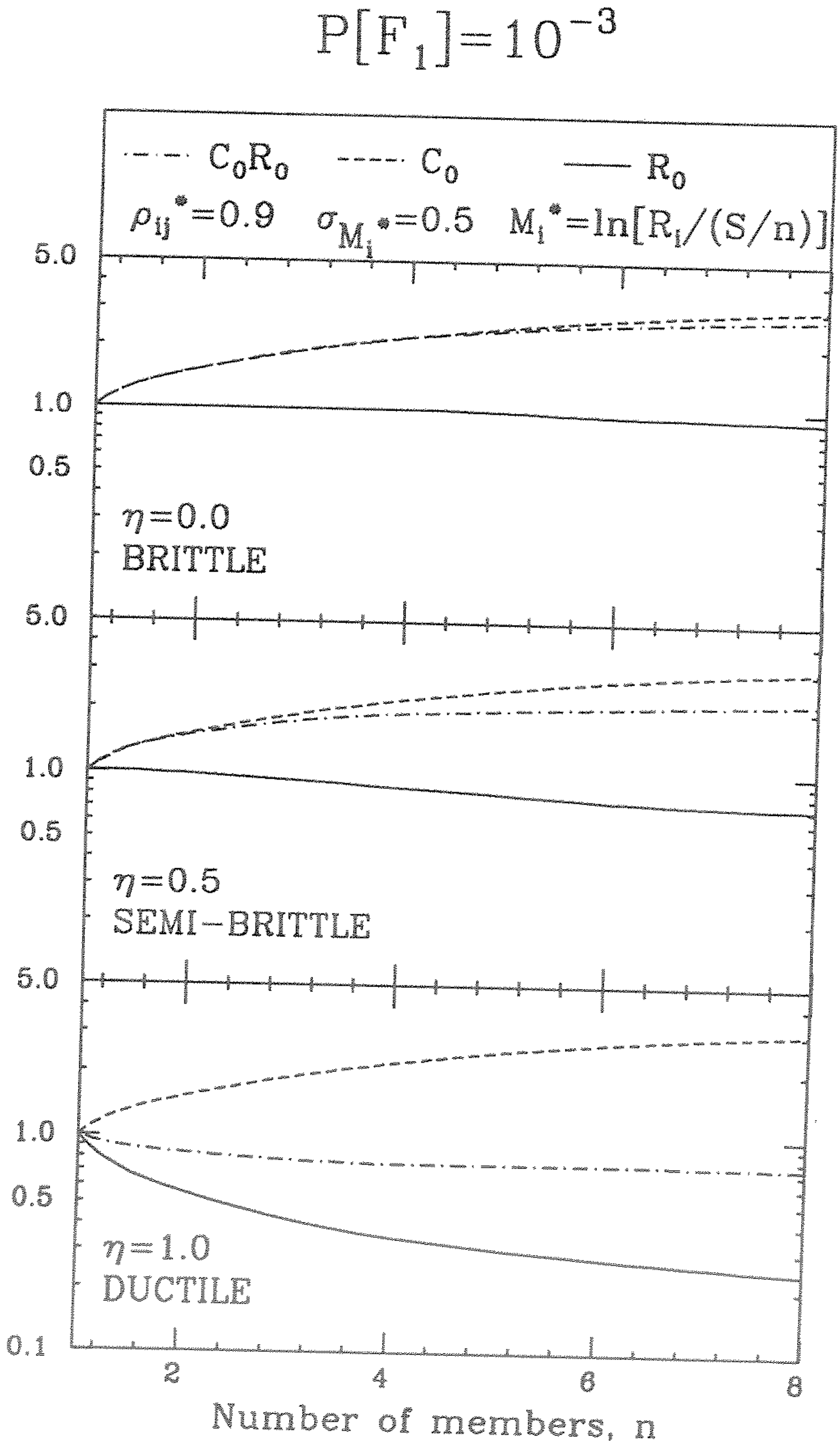

Fig. 3.13: C_o, R_o, and C_oR_o – **Complexity, Redundancy, and Net System Factor for Ideal Parallel Systems.** Lognormally-distributed resistances and load with parameters shown. Mean resistances specified to make $P[F_1]$ the same for all structures.

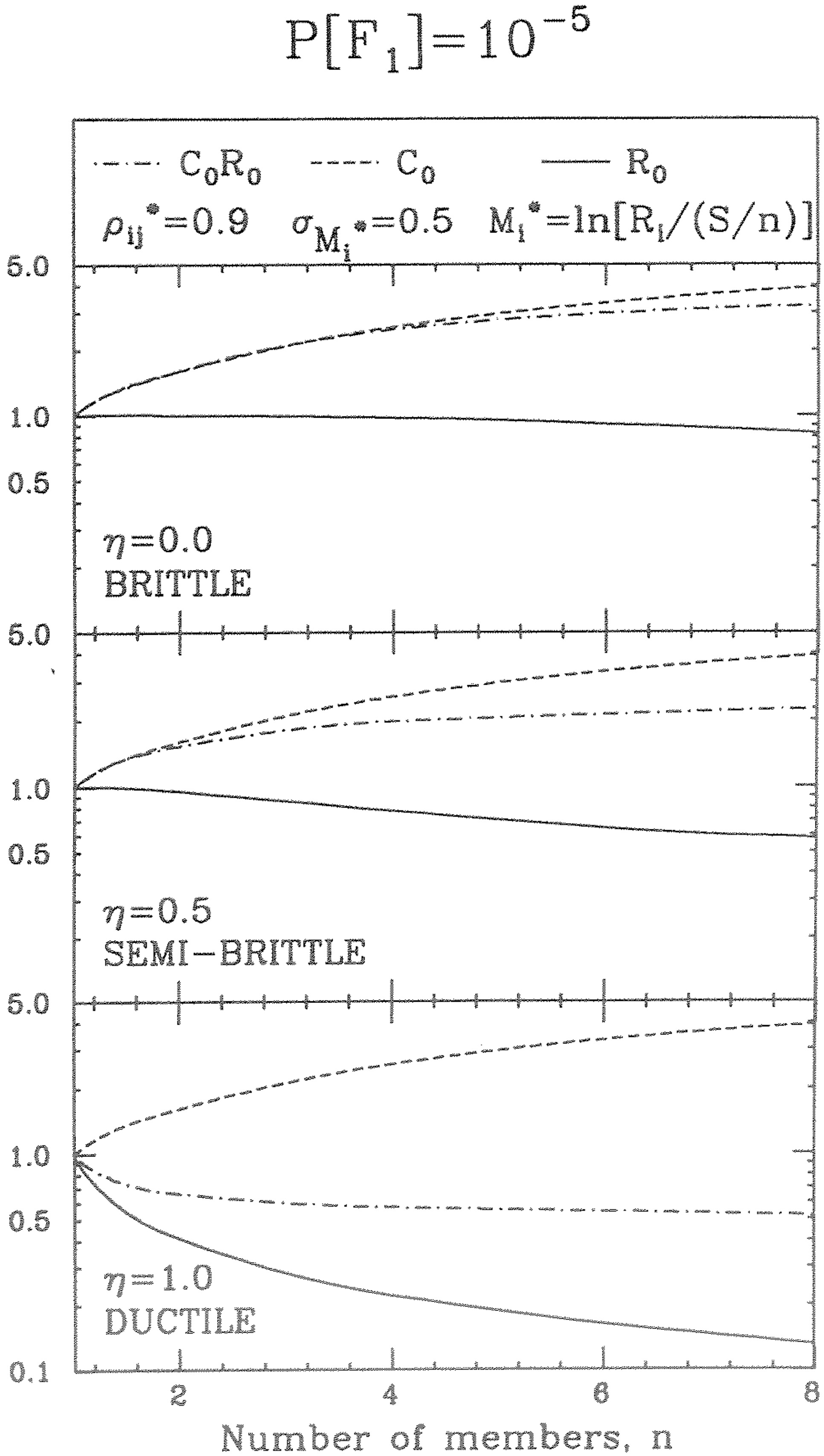

Fig. 3.14: C_0, R_0, and C_0R_0 — **Complexity, Redundancy, and Net System Factor for Ideal Parallel Systems.** Lognormally-distributed resistances and load with parameters shown. Mean resistances specified to make $P[F_1]$ the same for all structures.

Chapter 4

SOME ILLUSTRATIONS OF STRUCTURAL SYSTEMS RELIABILITY ANALYSIS

The purpose of this chapter is to illustrate the application of structural systems reliability analysis through two case studies. In addition we begin to gain insight through these examples into the kinds of questions and what kinds of results the engineer may ask and receive from systems reliability.

4.1 Steel Jacket Example.

This example has been presented in some detail in a companion project report entitled "A Systems Reliability Case Study of an Eight-Leg Steel Jacket Platform" *(Nordal et al, 1987)*. This study was conducted by Mr. Harald Nordal of Statoil and by the investigators of this project, with the cooperation of several of the technical representatives of the project. The FAILUR program developed by Dr. Yves Guenard *(1984)* was used to conduct the reliability analysis. The structure is the same one designed and studied previously by Lloyd and Clawson *(1983)*, deterministically but non-linearly. We are concerned with obtaining an estimate of the reliability of this structural system when subjected to extreme wave loads. The broader objective is to explore the utility of structural systems reliability with respect to answering questions about the configuration of the structure (e.g., X-bracing vs K-bracing), member sizing criteria (e.g., horizontal bracing), robustness, and sensitivity to various environmental and methodological assumptions.

The structure, shown in Fig. 4.1, was modelled as a truss with loads applied at the nodes. The nodal forces were estimated by applying to the truss a 100-year design wave of 63-foot height and using first-order wave theory. The capacities of the members in tension and compression were estimated by conventional formulas adjusted for any bias they may contain and given appropriate coefficients of variation. The probability distribution on the loading random variable was chosen from current API practice as represented in Moses *(1986)* for the Gulf of Mexico and on the basis of the judgement of the Norwegian engineers for the North Sea environment. (With the coefficient of variation and 100-year value specified the median of the assumed log-normal random variable follows immediately.) Post-failure behavior of the structural members was represented by the two-state semi-brittle members shown in Fig. 3.5 with ductility coefficient

η ranging from 0.4 to 1. The value of 1 was adopted for tension members and the value to be used for compression members was the subject of a sensitivity study.

Results. The estimated system failure probability for the Gulf-of-Mexico-located X-braced platform is 1×10^{-5}. The probability of failure of the most-likely-to-fail (MLTF) member is considerably higher, 4×10^{-5}. That is, the system as a whole is found to have a failure probability of about 25% of that of the MLTF member. In different words, given that the MLTF member is overloaded in the intact structure, the probability of system failure is only 25%. In the terms of Chapter 3 the product $C_0 \cdot R_0$ is 0.25. The probability that some member, i.e., at least one, is overstressed in the intact structure is 11×10^{-5} or about 3 times larger than that of the MLTF member. In the terms introduced in Chapter 3, this implies that the factor C_0 is equal to 3 for this structure. Dividing this factor into 0.25 we conclude that the R_0 redundancy factor for this structural system is 0.08.

Given the large number of statically redundant members in the system (Fig. 4.1), we must conclude that the coefficient of correlation among the member-failure margins is quite large. This is confirmed by observing the relatively large coefficient of variation of the applied base shear (the random variable which serves to scale the spatial load distribution); its coefficient of variation is 0.37*, while the member capacities have coefficients of variation in the 10-15% range. The implied correlation coefficient among (log) member-failure margins (Eq. 3.7) is in the order of 90%. Inspection of the probabilities associated with various branches and paths on the failure tree for this particular structure shows that the most likely member failures are in the vertical X-bracing of the bays at the second level of the platform. The MLTF member is in fact a compression brace in one of these vertical X-braces. Given failure of this member, the conditional probability of failure of the subsequent most-likely-to-fail member (namely a compression brace in an adjacent bent) is more than 30%. Looking at the several most likely failure paths, one concludes that the general failure process in this structure is almost certain to be a gradual deterioration of the vertical bracing system in this single story. The structure is well-balanced. There are several members with comparable intact-structure

*This number contains a large contribution from modeling (wave height to base shear and/or member force) uncertainty which is treated here in a simplified manner that may exaggerate the correlation between margins.

failure probabilities, the conditional probabilities of subsequent failures are relatively large, and there are many failure paths with probabilities comparable to that of the most likely failure path. Most represent minor perturbations of the order of failing of these third-level bracing members. Therefore their unions are highly overlapping and it is not surprising that the probability of failure of the system (the union of these paths) is little different from that of the most likely failure path.

Site Studies and Parameter Variations. If we change the location of the structure to the North Sea where the load coefficient of variation is smaller (in the study 23% was used in place of 37%), and adjust the median wave height accordingly (such that the 100 year wave is still the 63-foot design wave), then we find of course a much safer system. The system failure probability now is calculated to be 1×10^{-9}. The MLTF member has a failure probability which is much higher than the system value, namely 1×10^{-7}, implying a $C_0 R_0$ of 10^{-2}, lower (better) than that found in the Gulf of Mexico structure. This coincides with the conclusions drawn in Section 3.3. The value of C_0 for this structure, however, is little changed, implying that the R_0 term itself is approximately 0.003 for this structure, or more than an order of magnitude lower than that observed in the Gulf of Mexico structure.

Here we find that if we place two identical structures in two different environmental surroundings with the same 100-year wave, the structure which is in absolute terms more reliable (due to the less hazardous, lower COV loading environment) has redundancy in probabilistic terms larger than that of the less reliable system. The C_0 term is virtually unchanged implying that the net system effect $C_0 \cdot R_0$ follows closely the behavior of R_0. Recall that R_0 is the conditional failure probability of the system given that at least one member in the intact structure is overloaded. The apparent redundancy is larger in the North Sea environment, where the load COV is smaller, because it is less likely that a load which is adequate to fail one member will be large enough to also fail subsequent members. These conclusions are based, recall, on the assumption that the structures are designed for the same 100-year wave load.

K- versus X-Bracing. Next we sought to determine the impact of changing the configuration of the vertical bracing system to K-braced to begin to understand the impact of this "locally" statically determinate bracing system upon the system-level reliability. As is discussed in detail in the companion report, the choice of rules for arriving at a structure with comparably sized K-braced members was based on current member-sizing practice in the industry; this rule did not lead to comparable first-member failures probabilities and it is therefore somewhat difficult to make the comparisons as cleanly as one would like in probabilistic terms. Having re-configured and re-sized the structural bracing system, the calculated failure probability of the MLTF member for Gulf of Mexico environmental conditions was 1.1×10^{-3} or nearly two orders of magnitude larger than that of the X-braced system*; however our primary focus here is on the impact on the system as a whole and specifically on its redundancy. The system failure probability was calculated to be 1.3×10^{-3} implying a $C_0 \cdot R_0$ system factor slightly greater than 1: 1.2. The C_0 term is little changed from that in the X-braced system, here about 1.8, implying that the redundancy factor of R_0 is 0.65 in this structure. Comparing and contrasting these numbers with those for the Gulf of Mexico X-braced system, we conclude that the K-braced system has a system factor, $C_0 \cdot R_0$, and redundancy factor, R_0 about 3 to 4 times worse (higher) than that of the X-braced system.

There was approximately an order of magnitude decrease in the failure probability when the semi-brittle (η=0.4) compression member in the X-braced system was replaced by an elasto-plastic model (η=1). These models bound the correct strut model. Given the greater post-failure flexibility of the failed bent in the semi-brittle K-braced structure, the effect of changing to an elasto-plastic model should be even larger on this system. The conclusion is that the mechanical modeling limitations of the simple, binary semi-brittle member are important.

Horizontal Bracing Sizing. Next we consider the question of sizing the horizontal bracing. These members may play an important role in platform transportation and launching, but under the wave load they may be lightly stressed in the intact structure. An interesting question in the systems reliability context is what is the horizontal bracing members' role in the post-first-member-failure state of the system, when they may be needed to transfer lateral loads in the failed bent into parallel bents. As designed, the structure had the diagonal X-bracing members in the horizontal planes sized with a minimum thickness of

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*Careful scrutiny indicated that the difference was caused in part by the X-braced system's (deterministic) safety factor (with respect to the design 100-year wave load); this is induced by the fact that the critical bracing is stressed by gravity was well as lateral loads in the X-braced system. Also this study, based on test results, adopted for the X-braces "best estimates" ***kl*** values substantially smaller than those used for design, whereas they were less markedly reduced for the K braces.

3/8 inch and d/t ratio less than 60. As anticipated, in the intact structure the probabilities of failure of these members were many orders of magnitude smaller than those of the vertical X-bracing members. The forces in these horizontal-bracing members and their failure probabilities increased markedly, however, when failure in an adjacent vertical brace took place. In absolute terms these values were still small, however. This study concluded that one could completely eliminate one of the two members in the horizontal X-braces, before these members would begin to play an important probabilistic role in any of the failure sequences. In addition, when the case (see below) of a damaged member was considered, it was found that a sequence of failures involving the horizontal diagonals next to the damaged bent appeared among those that were the most likely. Therefore, once the X-configuration was replaced by a simple diagonal, the horizontal bracing (the sizing philosophy) could not be much smaller without impacting significantly on the robustness of the system with respect to damaged vertical bracing members.

This apparent (horizontal bracing reduction) conclusion must be tempered by the limitations of the illustrative example; the structure was not designed nor analyzed for other than wave loads nor did we analyze it for wave loadings in the broadside direction. This is a good example, however, of a situation in which both structural analysis in the post-failure range and systems reliability analysis provide quantitative guidance and answers that are difficult to assess by more familiar methods.

Robustness. The final study involves the robustness of the system with respect to loss of critical members due to some unspecified exogenous cause. When a third-level bracing member in a vertical bent in the K-braced system is removed, the failure probability of the system increases almost two orders of magnitude to 5×10^{-2}, failure being virtually assured when the wave is large enough to exceed the MLTF member capacity in any of the remaining bents. In contrast, for the X-braced system, if one critical tension member is removed the failure probability of the system increases only about one order of magnitude to 2×10^{-4} from 1×10^{-5}. The robustness of the X-braced system is better with respect to the loss of a critical bracing member. The redundancy factor, $\bar{R}$, for this damaged system has increased from an intact-structure value of about 0.08 to 0.7, reflecting the loss of redundancy in the system as a whole. The $\bar{C}$ value has decreased from about 3 to about 1 reflecting the relatively high stress level in a single member, the compression member remaining from the original X.

Unstated, however, is the assumption in the preceding results that the capacity of the adjacent compression member in the X-brace would not be reduced by the loss of the tension member. This is likely to be an unrealistic model of the X-bracing after the damage-causing event. Assuming, in contrast, that the loss of the tension member reduces as well the capacity of the companion compression member through the implied increase in the unbraced length (a loss of approximately 20% in axial capacity), one finds a failure probability for the system of approximately 3×10^{-4} or some 30 times that of the intact structure. This number is comparable to that (60) observed in the K-braced system implying that the X-braced system is only a little more robust with respect to loss of this particular (most) critical member than is the K-braced system to loss of a diagonal. Assuming the damage is caused by a falling object or boat collision, the X-braced system may, however, be less likely to lose its most critical member than is the K-braced, which is certain given loss of a bracing member at all. If one takes the pushover test literally, i.e., ignoring the alternating effect in real wave loads, then he might assume approximately a 50-50 chance of losing *either* the compression member *or* the tension member (but not both) in the X-braced case, the net robustness factor (Eq. 2.4) for a typical bracing bay becomes a factor of only 20, implying a somewhat more robust system than the K-braced. (The compression member failure implies only a factor of about 10 in increase in failure probability; 20 is just the weighted average of 10 and 30.)

Finally, if one considers the possibility that the X-braced system will be damaged to the extent that *both* the compression *and* tension member are lost, one finds an increase in the failure probability of the entire structure of a factor of about 65, virtually identical to the robustness of the K-braced system with respect to the loss of a brace.

Conclusions. This particular case study structure may not be particularly representative of typical steel jacket structures. Therefore the results must be interpreted with caution. The structural model neglected important realistic design considerations that would have impacted the systems reliability results. Most specifically, there are no launch trusses in the structure. This is in contrast to typical platforms where some of the members whose size is governed by launch loads also contribute to the resistance under wave loads. These members should be available to provide precisely that degree of post-first-member-failure capacity needed by a highly redundant structure to not only "absorb" the first-member failure but also to permit still higher applied loads before failure of the system as a whole. In short, it should be expected that realistic steel jackets will demonstrate much higher system and redundancy factors (C_0R_0 and R_0) than this particular example platform.

This study shows that there is important, relevant information to be gained from reliability analysis of structural systems. This information may include comparisons of different structural configurations with respect to their absolute reliability and with respect to their redundancy and their robustness. It is also clear that the measures for making these comparisons in probabilistic terms, such as the proposed robustness factor, require more study and more experience before we understand properly how to use and interpret this new tool.

4.2 Gravity Structure Example.

The reliability analysis of a concrete and steel gravity structure in some 130 meters of water was undertaken by researchers in the Netherlands *(Vrouwenvelder, 1985)*. This study is particularly interesting here from the perspective that, although the structure is complex, it was demonstrated that relatively simple mechanical models were adequate, that the systems analysis reduced to a rather simple form, and that even rather crude probabilistic methods (namely the mean-centered, first-order analysis) could yield useful insights into the behavior and reliability of the structural system. The structure is shown in Fig. 4.2 and its two-dimensional model in Fig. 4.3. The structure was loaded by wind, wave, current and dead load. The appropriate mean and standard deviations were assigned to these loads and to the several relevant capacity random variables, (concrete strength, reinforcing steel strength, pre-stressing steel strength, collision factor, etc.) As an initial approximation, the structure was assumed to behave in an elasto-plastic manner. Several possible failure mechanisms were identified and only one was found to be dominant, a simple side-sway mechanism involving hinges at the foundation and deck level, i.e., in the concrete and steel respectively. The formulations of the hinge capacity random variables recognized that they are functions of several random variables including the properties of the concrete and of the reinforcing, pre-stressing and deck steel. The means and variances of the capacities were calculated by approximate (mean-centered, first-order [e.g., *Cornell, 1968*]) methods from the corresponding moments of the material properties.

While the random variables in the problem have coefficients of variations ranging from a few percent to as much as thirty percent, the coefficient of variation of the capacity of the structural system failure mode as a whole was found to be only 11%. The so-called *safety index*, β, of this system was found to be 8.8. It is impossible to translate this reliability measure into a precise failure probability without more specific assumptions as to the distribution types in the problem. One can, however, compare this β-level with typical values encountered in member-reliability studies, where values of the order of 3 to 4 are not uncommon.

The structure was next reanalyzed using a sophisticated non-linear structural analysis program, STANIL, that includes explicitly both geometric and physical non-linearities. Two important limit states of system behavior were identified. The first is associated with "yield" strains in either the concrete or steel at any point; the second is associated with the ultimate strain level in the concrete at any point. The reliability assessment method was again simply mean-centered first-order analysis in which the necessary sensitivity coefficients or partial derivatives where calculated by straight forward numerical means, i.e., repeating the structural analysis for, first, the mean-centered variable values and for, then, unit-standard-deviation increases in each of the important random variables. The loading system was presumed to be proportional from zero for all of the several loads involved. The conclusion was that with respect to the yield-strain limit state only one of the two possible modes dominated, namely that of yield in the concrete. The β-level for this failure mode was 4.1 and the structure had an effective coefficient of variation of 25% in this "failure" mode. In contrast, with respect to ultimate strain, governed by ultimate strain concrete, the β-level was 8.2 and the coefficient of variation of 12%, agreeing very closely with the corresponding numbers found in the simple elasto-plastic mechanical representation of the system.

Perhaps the most interesting outcome of this study was a sensitivity analysis to ascertain the relative contributions of the uncertainty in the individual random variables in the problem to the uncertainty in each of the system failure modes as a whole. The conclusion was that an overwhelming portion of the uncertainty was contributed by the corrosion factor random variable. It was assumed that at the time at which the critical loading will occur there will have been significant corrosion in the reinforcing steel; the mean value of this reduction factor in the steel area was 0.7 and a coefficient of variation of 30% was assigned to this factor. (Based on the author's discussion one must assume that this coefficient of variation reflects to a large degree professional uncertainty as to whether or not this corrosion will take place at all, as opposed to case-to-case randomness.) The apparently somewhat controversial assumption about corrosion led to the conclusion that this corrosion effect contributed between 40% to 60% of the variability depending upon which of the models and which of the failure modes one was considering. The second most dominant random variable was that associated with the inertial coefficient, C_m, in the Morison equation. It was assigned a coefficient of variation of 25% to reflect the uncertainty in predicting basic structural load effects given the wave height. It contributed between 10% and 30% to the total variability in the problem depending upon in the model and failure mode. (Together these two random variables contributed a sum of about 70% to the total variability in all cases.) The only important environmental variables contribution was the wave height, which contributed some 5% of the total variability based on being assigned a 5% coefficient of variation. (This was the assumed COV for the maximum wave height in 50 years.) The reinforcing steel contributed several percent of the uncertainty of the ultimate strength mode, but, of course, did not figure significantly in the concrete-yield mode. (It had been assigned a coefficient of variation of 8%). The author states that the failure mode associated with yield or cracking of the concrete may be the most important for this system because of the implied corrosion effects.

Because only simple modes dominate, the system behaves effectively like a single member; all system factors C_0, R_0, and $C_0 \cdot R_0$ are unity.

While the systems behavior aspects of this problem tend to be relatively simple, due largely to the dominance of a particular failure mode or sequence, it demonstrates that we should be prepared to address rather complex structures with relatively simple reliability methods, when they are adequate for the job at hand, and that even these simple analyses can produce results of interest and significance to the engineer.

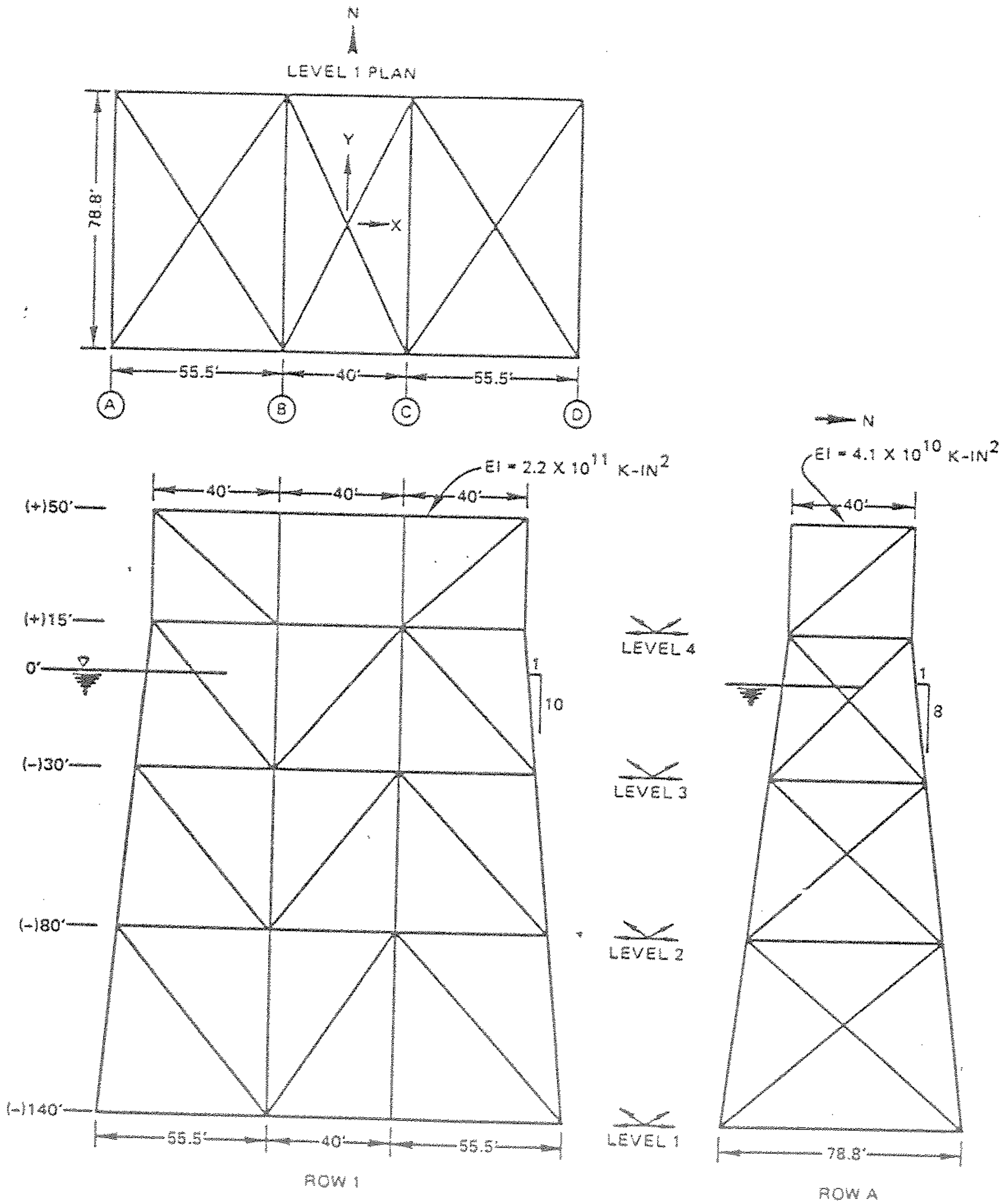

Fig. 4.1: Example Problem Structure.

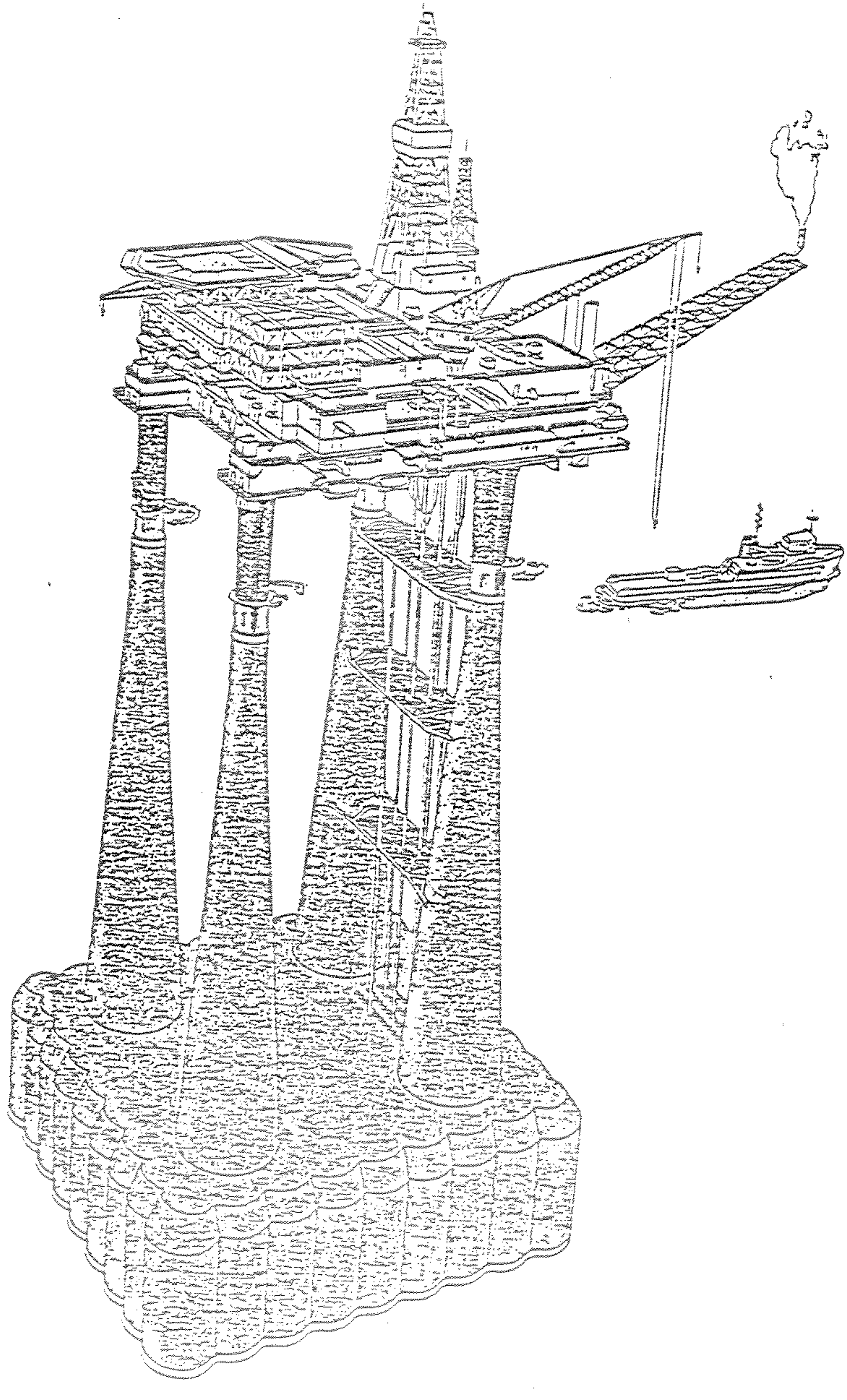

Fig. 4.2: Dunlin-A Gravity Platform Source: *Vrouwenvelder (1985).*

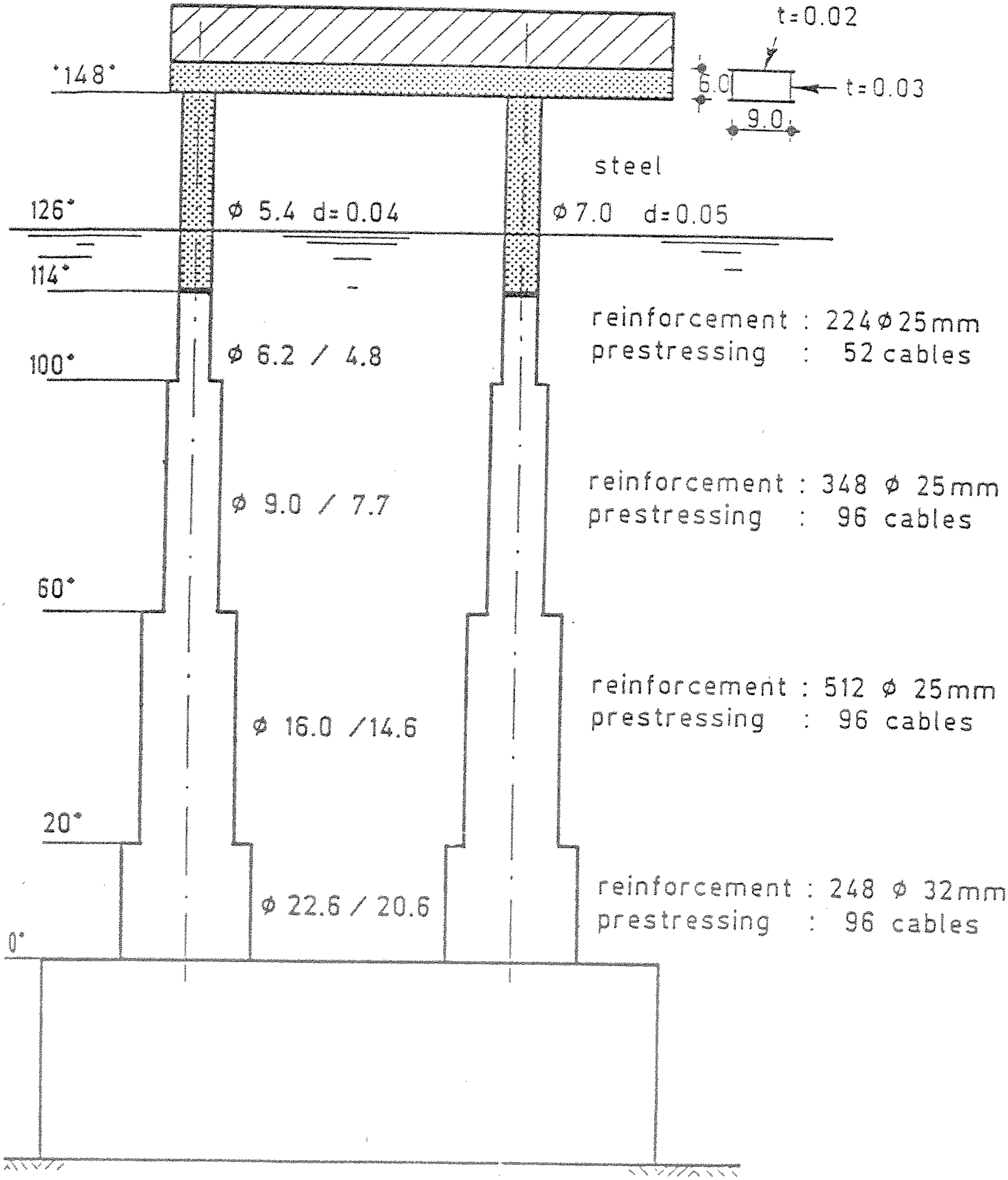

Fig. 4.3: Schematization of the Gravity Structure.
Source: *Vrouwenvelder (1985).*

Chapter 5

SYSTEM RELIABILITY NEEDS IN THE INDUSTRY

In this chapter we shall attempt to identify in general terms many of the types of problems and situations in which structural systems reliability would enhance the practice and operation of offshore engineering. Our ultimate objective will be to compare these needs with methods which are available to satisfy the needs. The appropriate methods will be discussed in Chapter 6 and in more detail in the companion report (*Karamchandani, 1987*).

As elsewhere in the report the focus in this chapter will be on structural systems reliability as distinct from member behavior and as distinct from member reliability. We shall focus also in this chapter on safety as distinct from availability. The word "availability" (sometimes, indeed, the word "reliability" itself) is used to mean the fraction of the economic life in which the system is available and functioning as intended. Many of the methods that we shall discuss here have been and are used in engineering practice to study availability as well as safety. Further, our focus will be on fixed platform structures as opposed to exploration and production facilities. Finally, the emphasis will be on failure of the structure, although we understand perfectly that operational procedures, accident prevention and many other portions of the process are equally important. We are, however, interested here in the potential interaction between the structure and elements of the design and operation. For example, as we saw in Chapter 2, we can define formally an interaction between the redundancy of the system, e.g., $\overline{R}_j$ and the inspection quality, $I^Q_{o,j}$ in Eq. 2.3.

We recognize that whether we are discussing members or systems, and whether we are discussing deterministic or reliability-based analysis that many aspects of the studies will have common characteristics. For example, we understand that there will be different levels of analysis resources and effort depending upon whether the problem is a comparative conceptual study, a feasibility study, a design and analysis, or an evaluation of an existing structure that is damaged. There will be in these various situations different levels of complexity in the structural modeling and analysis, the representation of the loads, and the details with which the problem is specified. Because our focus is on system safety, be it deterministic or probabilistic, we shall emphasize post-failure behavior of members and systems. Therefore, non-linear structural analysis methods will be used generally. In some cases, linear-elastic structural analysis, be it static or dynamic, may be adequate for the investigation of fatigue and crack growth and possibly the consideration of operational difficulties and costs implied by first-member failure.

We shall look below at three general categories of offshore industry needs. The first is the evaluation of new platform concepts and environments. The second is the improvement of existing platform concepts and design procedures. And the final area is the assessment of operating platforms.

5.1 Evaluation of New Concepts and Environments

The search for oil in deeper waters and in new environments such as the arctic have created an interest in new structural concepts for platforms. In virtually every case new system concepts are constructed from familiar materials and members, e.g., structural steel members, cables and stiffened shells (hulls). What is primarily new is how these members are configured into a system. In the same way that structural analysts are interested deterministically in how the new systems might behave, so too should structural systems reliability analysts be concerned about the reliability implications of the new configurations. Has redundancy been maintained? Has residual strength been maintained? Has robustness with respect to member or local failures been maintained? What is the relationship between the complexity of the system, its redundancy, and its robustness (as measured by C_0,R_0, and ROB_1, for example).

For new concepts, many of the systems reliability studies might be carried out at a relatively simple level. Perhaps, rather crude mechanical and systems models might be adequate. One would anticipate that the emphasis would be on *comparative* reliability, redundancy and robustness, the comparison being with more familiar existing platform concepts such as steel jackets. For this reason alone, one would want to perform systems reliability analysis of existing common platform types. The deep-water aspect of many of the novel structural concepts implies that new kinds of load combinations and recurrence probabilities may be of interest. This is different from what the industry is accustomed to. Deeper water implies the introduction of compliant structures with their less familiar and hence perhaps less confidently predictable behavior. Currents and wind play a larger role. And finally, it implies the strong role of structural dynamics. All of these elements have the implication that the relationships between members and the system, and between member safety and system safety may potentially be different from what we are accustomed to based on the industry's experience. This in turn implies the potential utility of structural systems reliability to combine and study these various issues.

Uncertainty. These novel environments and behaviors may imply the existence of a significant amount of uncertainty. We use the word "uncertainty" with care and precision here to distinguish it from randomness, as mentioned previously. Uncertainty can be reduced with increased information such as increased experience or more refined analysis and testing. System reliability promises to capture and display this uncertainty for analysis and decision making. Although not common practice yet in the offshore industry's reliability analyses, it is commonplace in nuclear-industry PRA practice to propagate through the system uncertainty separately from the randomness, and to display at the end of the study a statement about the uncertainty in the failure probability estimate.

For novel structural systems we would expect this uncertainty (almost by definition) to be greater than for existing systems. The importance of this uncertainty needs to be emphasized in novel-structure planning and decision making. Under what circumstances should one build test structures (e.g., the Navy's deep water moored semisubmersible or a less-than-full-depth prototype). Can novel structures' uncertainty be offset by a more intense inspection policy? Uncertainty's importance in novel structures also requires an analysis capability that permits the comparing and combining of uncertainties from different members and portions of the total system. It is necessary to update uncertainty (typically reducing it) as new information and data become available, e.g., test structure or prototype experience, from inspection reports of crack lengths, etc. The need is for an analysis capability that propagates the uncertainty in the elements of the problem, e.g., loading, structural components, etc. into the uncertainty in the total system behavior. Clearly, for example, a great deal of uncertainty can be tolerated in members and types of behavior which are not important to the overall systems reliability.

This novelty and uncertainty also suggest the need for careful consideration of the robustness of the system. Because experience is lacking with exactly how the system and environment will interact and because external inspection will be difficult in deeper water, it may be particularly important that one encourages platform systems that are damage tolerant. It may be useful to consider an analysis in which it is presumed that, one-by-one, each individual member or portion of the structural system has been damaged or eliminated for whatever reason, and then to analyze the reliability of the remaining damaged system. This is an example of the analysis of the system due to exogenous events in an *a priori* mode described in Chapter 2. The Quantitative Safety Goals promulgated by the Nuclear Regulatory Commission in recent years have included the requirement that one study the safety of the total plant system given that a core-melt accident has occurred for whatever reason. Such a requirement puts an emphasis on the damage and/or system impact mitigation capability of the containment structure, the spray systems, operators, etc. The progressive limit state (PLS) consideration in the Norwegian NPD regulations follows this same logic.

We understand that the systems reliability analysis should in many cases be extended beyond the pure structural behavior into operational and inspection policies. This may interact with the structure itself in this larger systems representation. Operational changes may reduce accidents (p_i) or the impact of accidents (should they happen) on the structural integrity ($p_{j|i}$) or both (p_{ij} in Chap. 2, Eq. 2.2). In a novel structure where there is no experience with its behavior inspection plays a much more important role; it reduces uncertainty. The long-term reliability of the system may be strongly dependent on the degree of inspection given a new structural detail such as the connection of a TLP tendon to its hull. Therefore, the inspection policy for a novel structure may be an important integral part of the total system safety analysis.

Many of the concepts of structural systems reliability contribute to or have been borrowed from the larger field of systems reliability. The successful practical application of PRA in the nuclear industry and elsewhere suggest that it can be a useful tool as well in the offshore industry when it explores the introduction of new production concepts such as subsea systems and/or new methods of pipeline construction and design. Structures are predominantly passive. Active system components (controls, valves, human beings, etc.) require, however, very similar systems analysis tools.

All of these elements of novel platform concepts suggest the importance of understanding well the strategy for providing redundancy and robustness at the systems level. The fact that system reliability decreases as complexity increases must be remembered. This observation favors simple solutions. The fact that system redundancy decreases if all elements of the structural system are in near-perfect balance under the dominant loads should be kept in mind. Alternate loadings such as those during the construction process may have a largely unappreciated, but important role in creating backup capacity to share the loads under extreme environmental conditions should one member fail. The importance of the availability of lightly stressed members cannot be ignored. System robustness may be improved by insuring that damage caused by, for example, ship collisions or operational accidents such as explosions be localized and that alternate load paths be provided. The ability to analyze the impacts of these system-level strategies upon the reliability of the novel platform should be beneficial to structural systems designers. The subtle role of randomness and relative degrees of randomness (e.g., in the load versus the capacity) as they impact element margin correlations and in turn systems reliability should be understood. Systems reliability provides the only tools to produce this level of understanding at the stage of novel concept design.

5.2 Refinement of Current Design Practice

Although the industry experience with most of the existing platform design concepts, most particularly the steel jacket structure, has been exemplary, the opportunity to reduce further the cost of such systems without impact upon their reliability may be important in the current economic environment. If one has the ability to analyze the structure's safety at the system level as distinct from the member level, it may permit the possibility of changes in current design practice. Examples include sizing rules for horizontal bracings, the impact of pile foundation redundancies, alternate bracing schemes (e.g., X versus K bracings), joint capacity sizing rules, etc. Systems reliability might also evaluate and help improve the current design practice procedures for evaluation of platform safety. Deterministic developments have already gone forward in seismic overload criteria, including post-member-failure analysis by static or dynamic means. The PLS concept in the NPD regulations is aimed at the robustness question. How should it be routinely implemented? Coupled with systems reliability one could analyze quantitatively the effectiveness of such criteria alternatives.

The preceding need suggests the requirement for effective cost-benefit-risk analysis possibly using formal mathematical programming or optimization procedures. Such techniques are in practice in the aircraft and aerospace industry on a deterministic basis and their application has been proposed in the probabilistic context, as well.

At the broader level, one can identify immediately the advantages of being able to trade off the cost of operation changes and/or inspection policies versus their impact upon system reliability. That impact might be upon accident occurrence rate, accident impact mitigation and even member sizing where fatigue and crack growth are critical.

The industry has already moved rapidly in the implementation of probability-based, load-and-resistance-factor and partial-factor design codes. The developments to date, however, have been limited to member- level considerations. Structural code development for decades has been sensitive to system-level questions; buckling criteria, for example, are separated into primary and secondary members; reinforced concrete codes recognize that ductility will permit moment distribution different from the simple elastic estimate; seismic Building codes distinguish between more brittle systems and more ductile systems. All of these considerations have, however, been included in the code on the basis of qualitative experience and judgment. There is need to complement the current member-based code development with systems reliability analysis. It could provide the quantitative information to determine the circumstances under which conventional member-level criteria should be modified to reflect system implications. One can envision a system factor contained within the resistance or load factors of current probability-based design code formats. We know from our discussion in Chapter 3 that the failure probability level of a system depends upon system complexity and system redundancy. A system factor in a code would penalize a more complex and less redundant system and, vice-versa, reward the less complex and more redundant system. How to practically develop the values of such a system factor and as a function of what easily identified system and load characteristics is a major open question today.

Finally, the potential advantages of an "information-sensitive" code should be considered. These codes provide for the explicit dependence of load factors, for example, on the amount of information available, reducing the factor as site-specific data is collected. Such a factor must make use of formal information updating procedures (*Kilcup, 1985*).

5.3 Assessment of Operating Platforms

Many industries have learned that evaluation of existing structures is one of the major potential areas of benefit from reliability and specifically from systems reliability. Examples include the dam safety management program in the U.S., re-evaluation of the inventory of existing highway bridges, and retrofitting problems in the nuclear power plants industry where new experience and industry accidents have enforced the need to re-evaluate operating plants. The simplicity and conservatism of upper-bounding, deterministic design concepts (e.g., the probable maximum earthquake) may be tolerable when one is preparing the design of these systems. With re-evaluation there may not be the margin for such simplicity. Further, there is less professional experience with and fewer time-tested procedures for re-evaluation, re-assessment, and damage impact prediction than with new design. All of these areas suggest the need for systems reliability application.

Early Generation Platforms. With the maturity of the offshore industry has come the question of dealing with older offshore structures. Whether it is a single structure or a group of structures, the question of the assessment of the current status of the safety of an old structure is a difficult problem. The level of information may be low and the cost of obtaining that information may be high and unwarranted. The status of the capacities of various members may be highly uncertain. Other new issues may exist in the systems reliability question of old structures (and more generally any existing, operating platform). For example, all the members in the system have been exposed to a comparable environment, e.g., comparable corrosion levels implying high levels of member-to-member correlation in the capacity reduction due to corrosion. There are certain aspects of the problem that are better known for existing structures than for structures at the time of design or concept development. For example, the degree of marine growth and more generally, the characteristics of the local marine environment including wind and wave levels are better known late in the life of the structure than before its construction. Wave-structure interaction can be monitored and a major uncertainty reduced. Reliability analysis and more specifically systems reliability analysis can take advantage of these reductions in uncertainty to demonstrate comparable safety levels late in the life of the system compared with those when the designs were only on paper. These questions of life extension of existing structures represent delicate balancing of uncertainty and reliability. The availability of formal quantitative methods should enhance significantly engineering judgment.

Re-Assessments. Somewhat similar questions arise when an existing facility must be re-evaluated in the light of new information about the behavior of such structure or new information about the loading environment. Systems reliability analysis has proved a useful tool in such cases. The introduction of novel systems with as yet unobserved behavior may lead to an increased frequency of such re-evaluations in this industry, as well. Systems reliability can often be called upon to find levels of system capacity and system behavior that were not exploited during the original design when less sophisticated methods were justified.

A somewhat similar re-evaluation may be required when the owner wishes to upgrade the use of an existing platform. For example, he may wish to increase the working loads on the operating deck. Again, it may be possible to use more advanced analysis methods to demonstrate that the reliability of the system as a whole will be adequte if the new operation is permitted. In particular, one can again in this situation take advantage of the reduction of uncertainty that has taken placed in certain elements of the problem since the structure has passed from a paper design to an in-place reality. If this approach is not successful, the methods may be used to minimize the cost of alteration to the structure to achieve the upgrade.

Damaged Structures. Undoubtedly, one of the most important needs in the industry is to assess the implications of damage to an offshore structure. This evaluation may be needed *a priori* for example, when one is considering alternative inspection policies as discussed above. More commonly, it is necessary *a posteriori* when the occurrence of damage to the system leads to the question of whether one needs to abandon the platform, repair or alter it (now or later), or simply watch it through more intense inspection. The main question is what has been the impact of the damage on the reliability of the structural system. A formulation of this question was discussed in Chapter 2. Clearly, some members are more important to the reliability, redundancy and robustness of the structural system than others. The availability of members which are relatively oversized for environmental loads may be exploited at the systems level to demonstrate adequate residual strength and safety. Such a damaged-structure review may involve a very detailed analysis and information collection with respect to a localized section of the structure; consider, for example, a case of a throughgoing crack in the joint of a jacket. Systems reliability including the implications of the increased information, might be effective in demonstrating continued system operation without significant system deterioration. If repair or modification is necessary, the effectiveness of alternative repair strategies with respect to the system's safety can potentially be better evaluated through reliability based analysis.

5.4 Summary

Reviewing all of these classes of needs at the various stages in the life of the platform in which they might apply, we come to the conclusion that the requirements upon structural systems reliability in the future are very broad and very heavy.

From a mechanical modeling point of view it is clear that there should be systems reliability procedures capable of paralleling all the deterministic structural mechanical analysis levels. This ranges from simplified mechanical models through typical design level analysis to advanced specialized studies. The mechanical modeling problems include: structures and soils; statics and dynamics, both linear and non-linear; high- and low-cycle fatigue; and both discrete and continuous systems. We can see the need for both typical truss and frame structures as well as continuous models of structures, and even finite element analysis representations.

Similarly, a variety of loads modeling needs exist. They include many load types: wave, current, wind, seismic and ice, at least. They include both the static and dynamic frequency ranges for structures with a broad frequency range. They include short-term, e.g., within wave and within storm, and long-term needs, e.g., storm- to-storm and year-to-year.

One can see the need for specialized procedures for routine application to relatively narrow classes of structures, e.g., static analysis of steel jackets, and also general purpose analysis techniques that could be applied to broad classes of problems. The former should be accessible to a broad group of knowledgeable engineers, the latter should perhaps be available to those with more specialized training but their applications may be to both simple, specialized problems and to large scale detailed problems.

In addition to the structural and load modeling questions, there is an evident need for procedures that include inspection and operation issues as well as accidents. This implies the ability to represent human factors and policy assessments. Cost-benefit and/or optimization procedures would be useful in a broad set of the applications needs discussed above.

A recurring theme in the three sections above was uncertainty. There is a need to assess uncertainty quantitatively and accurately, and also to update it in the face of new information. The treatment of this uncertainty should be carried along in parallel with that of the randomness in the loading and material properties.

Chapter 6

STRUCTURAL SYSTEMS RELIABILITY METHODS: OVERVIEW

This chapter addresses the methods currently available to compute the reliability of structural systems. It is worth repeating that our focus continues to be on the issues that make structural systems reliability unique. As with member-reliability analysis, there are many questions such as computational techniques for the analysis of large numbers of random variables, treatment of uncertainty as distinct from randomness, random vibrations, fatigue, etc., that must also be addressed in the systems reliability problem. In particular, we have in systems reliability the need for efficient computation of the probability of events associated with the value of a function of a large number of random variables, e.g., a member safety margin being negative (or member failure occurs). We also shall, as the methods advance to proper treatment of multiple loads and of dynamic responses, have need for stochastic process analysis. As we have discussed, questions that become specific or particularly critical to systems as opposed to members include the specification of post-failure behavior of members and non-linear analysis of large structures, and the (probabilistic) treatment of intersections and unions of failure events associated with individual members or components, both in the intact structure and in subsequent "partially-failed" states of the system. Indeed, the relationship between the states of the members and the state of the system as a whole is a fundamental concept in systems reliability.

The analysis of the reliability of a structural system requires three major steps. The first is the specification of the mechanical behavior and loading models. The second is the development of the relationships among events and among the random variables in the problem. And the third is the computational step. Each of these steps has its particular features but each also has the characteristic that for purposes of engineering analysis simplification and approximation may be made. The first step, the specification of the mechanical model and load model may appear obvious, yet, it has been the author's experience in reviewing literature in systems reliability in the past year that this particular step has not in all cases been given proper care. The precise mechanical and loading assumptions being made by the investigator of a problem may not always have been stated. The second step was illustrated in Chapter 3 for some particular cases; for example, the truss in Figure 3.1. The event "system failure" for a statically determinate structure was found to be the union of member failure events (Eq. 3.2) and each of these in turn could be written in terms of an event involving a function of two random variables, namely the load level and the capacity of that particular member. We saw in Section 3.4 that the events specification for the sequence of failures in a statically indeterminate structure could be simplified, Eq. 3.18, for the purpose of facilitating their formulation (and computation). The actual computational and numerical schemes associated with step three, with calculating, for example, the probability of the union of the set of intersections associated with an equation such as Eq. 3.19 are not trivial; suffice it to say, the difficulty of doing so well has only recently been surmounted and the computation not unexpectedly involves its own set of approximations. The engineer is familiar with making mechanical modeling and loading approximations for purposes of simplifying his analysis, continuously making judgments as to their adequacy for his purpose. Precisely the same kind of reasoning must be made with respect to the treatment of the second and third steps, of event specification and computation, in system reliability analysis.

6.1 Discrete-State Systems

In Chapter 3 we studied the formulation and the behavior of systems that could be represented as shown in Figures 3.1 and 3.3. They are characterized by a single, scalar random variable to represent the load or load system and by members whose capacities can each be described by a single random variable. Further, the structures were assumed to be linear and elastic when intact and the post-failure behavior of the members was characterized by the force-deformation diagrams in Fig. 3.5. Their form implies that, as the load is increased monotonically from zero to its final (random) value S, a failed member can simply be dropped from the structure, being replaced by its post-failure force, ηR, and a new ("partially failed") *linear* structure analyzed*. We saw that for this model the structure could be represented in a tree-form, e.g., Fig. 3.4, in which each path through the tree to the system-failure state was associated

* The implicit analysis assumptions of this popular mechanical model in systems reliability are a little more subtle than this; see Karamchandani, 1987, Section 3.1. They involve, for example, load- versus displacement-control, failure of subsequent members during the unloading of the failing member, etc.

with a particular sequence of member failures. For this class of problems we were able to represent the system-failure event in terms of a union of intersection of member-failure events described by Eq. 3.8. Each event in the right-hand side of that equation represents a branch in the ordered sequence of member failures. Subsequently we saw how these events could in turn be represented in terms of safety margin variables, e.g, Eq. 3.9, which in turn could be represented by the load and member-capacity random variables such as Eqs. 3.10 through 3.12. We shall not discuss the computation of the probabilities of these events beyond recalling (Section 3.4) that that computation is simplified if the events themselves are simplified by ignoring what we call the ordering events, (such as $(M_1^0 < M_2^0)$ in Eq. 3.9), in the individual branch definitions.

This particular class of structural reliability problems can be characterized more generally by: the load can be represented by a single random variable, even though it may reflect (scale) a spatially distributed pattern of loadings on joints or members; the load is assumed to increase monotonically from zero to the random value S; the individual members have capacities represented by a single load effect (e.g., axial force or moment); the mechanical behavior of a member can be represented by two states, referred to as the *intact* and the *failed state*. (In the latter state the member has no stiffness). The implication is that the system itself has a finite set of states, each represented by a combination of binary states of all the members. It is further implied that there is a finite set of (ordered) sequences of member-state changes or failure *sequences* by which the system can pass from its initial intact state to its final failed state. In different words, the failure sequence tree (Fig. 3.4) has a finite number of branches at each node and, therefore, a finite number of paths through the tree.

Searches. Perhaps the most effective current available methods for large scale structural systems reliability are those that address this particular class of structural reliability problems. It has been studied by many authors (e.g., *Murotsu, 1983; Moses, 1982; Bennett and Ang, 1986; Thoft-Christiansen, 1984; Melchers and Tang, 1984; Guenard, 1984; Bjerager, 1984;* etc.). Several computer programs are available to estimate the reliability of a system which can be modeled in this way. The computation procedures themselves differ from author to author. They virtually all, however, have the following general characteristics. All the authors replace (implicitly, at least) strict failure sequences by the failure path approximation (Section 3.4). They then search through the failure (path) tree with the objective of identifying one or a limited number of the *dominant paths* from among the potentially very large number of failure paths that pass through the tree to the system failure state. A dominant sequence or path is one with a relatively large probability of occurrence. One wants to restrict the probability computations to only one (or a small number) of the failure sequences; the calculation of the probability of the intersection of the sequence of member failures and then of the union of those intersections may be relatively costly and only recently was accurately estimated.

Many different schemes are available for more or less effective searches through the tree to identify the more likely failure paths of this structural system. The simplest method simply replaces all random variables by their mean values and conducts a deterministic analysis, incrementing the load from zero, identifying the (unique) sequence of member failures, and calling this sequence the dominant sequence (e.g., *Moses and Stahl, 1978*). More complex and effective schemes attempt to identify at each step (i.e., at each successive node in the tree), the member which is most likely to fail next. They pass on that chosen branch to the next node, repeat the process step until the system failure state is reached, and collect this sequence of member failures as the dominant path. The estimation of "most likely" at each node is based on a marginal calculation (i.e., $P[M_i^{(k)} \leq 0]$), on a ("first-member-and-next-member") double intersection (e.g., *Murotsu, 1983*), or on a complete statement such as the conditional probability of failure of the next member failure given all the preceding member failures *(Guenard, 1984)*. In contrast to Guenard's formal branch-and-bound (look-back-and-ahead) algorithm that guarantees that the most likely path is found, the search techniques are largely ad hoc, selecting as the "dominant path" a path of unproved superiority. This benefit may or may not be important to the final accuracy.

Intersections and Unions. Once a proposed dominant sequence of failures (or set of sequences) has been identified, the next objective is to write the statement defining the intersection of the individual events in this sequence in terms of the random variables involved in the sequence of member failures, e.g., the load random variable and the several involved-member capacities. If the structural system happens to have been represented as elasto-plastic, the last of the member-failure events corresponds to a failure mechanism, whose margin can be expressed in terms of a linear combination of resistance and load random variables. In short, the search has identified a (probabilistically) *dominant collapse mechanism*. The intersection/path to this mechanism can be ignored. In fact, historically, the tree searches through complete, ordered failure sequences were introduced simply to facilitate identifying stochastically dominant plastic collapse mechanisms from among the many such mechanisms that exist for a large structure. (This fact, coupled with the relative insensitivity of both linear and elasto-plastic structural analysis to the details of "path" by which final load levels are achieved, may explain some of the lack of precision one can find in the structural systems reliability literature, particularly with respect to statement of mechanical modeling assumptions and the limitations of the failure path approximation.)

For other than the ductile case, however, the entire set of intersections (i.e., the entire sequence of member failures, not just the last) must in principle be considered. In recent years it has become possible to calculate approximately the intersection of a number of events each involving a function of a vector of random variables, i.e., the probability of a failure path. The successive improvements of this scheme are identified primarily with the Technical University in Munich *(Hohenbichler and Rackwitz, 1983; Hohenbichler et al, 1985)*. We do not intend here to describe the computational techniques involved, they are well documented elsewhere (e.g., *Madsen, et al, 1986)*. Finally, many of the more recent methods for solving discrete systems under scalar loadings have searched for not a single dominant failure sequence, but a set of two or more such failure paths. The final step is then to calculate the probability of the union of this identified set of sequences, which are, in turn, intersections of member-failure events. Here again recent computational schemes have made it possible to obtain quite accurate and efficient estimates of the probability of the union of intersections of events that are in turn functions of a vector of random variables (e.g., *Ditlevsen, 1979; Hohenbichler et al, 1985; Madsen et al, 1986)*.

Multiple Loads. The class of discrete systems reliability analysis can be extended to multiple random loads, at least under certain limitations. Many real problems involve two or more random loads, e.g., gravity loads (dead and operational deck loads) and (largely) horizontal wave loads. The simplest model of such a problem, one often seen in the reliability literature, has a single random variable scaling a (deterministic) spatial pattern representing each load type. The loads are said to be time invariant and simultaneously acting.

Unfortunately this is an incomplete specification for systems reliability. One must specify the *load path*, a description of the relative rates at which the loads will be applied. For example, one must state that the two (random) load levels will be achieved by proportionally increasing them from zero. Or the dead load was applied first and then the wave load. Although only recently well appreciated, the load path question is essential; it is a simple matter to construct realistic structural problems in which the final values of the (two, say) load values are the same, but depending on the load path the structure either fails or does not. Simple stability and uplift problems are classic examples; members loaded in tension by one load and in compression by the other are common in large systems (e.g., the tension member in a vertical X-brace).

For the structural behavior represented by the model in this section, the analysis techniques can be applied without modification provided it is specified that the load path is proportional from zero *(Karamchandani, 1987)*. This specification insures that in a gross sense the loading is monotonically increasing. As individual members fail, of course, this condition may not hold at the individual member level. This is true even under a single load, of course. As mentioned above, the mechanical model and load event assumptions must allow for this possibility, if only by stating conditions under which the analysis may not strictly apply (e.g., elastic unloading of already "failed" members, failure of additional members during the "unloading" of a failing brittle member, etc.).

Another way in which the model may be used is to specify a random set of load paths, each of which can be characterized in its intensity by a single random variable. Each path might be a (one-to-one) function such as $S_2=h_i(S_1)$ with S_1 specifying the load level. After substitution of S_2 by $h_i(S_1)$ only the scalar S_1 would appear in the reliability analysis. The analysis* would produce a system failure probability for each path in the set, $i=1,2,\ldots s$. The final step is a weighting of the failure probabilities obtained by the likelihood of each path function. Note that this approach may be a natural one in cases when different loading scenarios are envisioned. It could even be used, rather awkwardly, to reproduce the case of proportional loading to a random point (S_1,S_2) in load space by transforming the joint distribution of S_1 and S_2 into polar coordinates (θ,R), discretizing into a set of directions $\theta_1,\theta_2,\ldots\theta_s$, and representing the load intensity by the length R with distribution, $f_{R|\theta_i}$. For each θ_i, the two loads would be substituted in the analysis by $S_1=R\cos\theta_i$ and $S_2=R\sin\theta_i$, again reducing each systems analysis to a scalar load problem.

* Strictly only the failure path method.

Finally, even these schemes will not work for even such a simple load path as gravity loads, S_1, applied first and lateral loads, S_2 afterwards. It is in principle possible to extend the same kind of tree scheme to this problem by adding *success branches* ("no further member failures") to each node. Then one applies S_1, first, and out of each success-branch end node appends a second tree associated with next incrementing S_2. The size of the total tree expands rapidly of course.

In short, even so-called time-invariant, simultaneous multiple load cases must introduce time implicitly through the specification of the load path. Explicit treatment of (static) time varying multiple loads on discrete systems has recently begun to receive sustained attention *(Guers and Rackwitz, 1986; Wen and Chen, 1986)* via *outcrossing* analysis. One can visualize the trace with respect to time of $(S_1(t), S_2(t))$, starting from the origin, passing from the safe domain across the particular segment of an overall intact-system g^0-function corresponding to first-failure in Member k, followed by an updated, damaged system g-function, g^k, etc. The tree structure remains; the analysis of each member-failure event (each branch) increases significantly.

Conclusions. Many of the methods of analysis of this class of discrete systems are discussed in much more detail in Chapter 3 of the companion report by Karamchandani *(1987)*. Two of the more popular methods are denoted Member Replacement Method and Incremental Load Method; they are effectively equivalent in many respects. As they have been applied in the literature, the former method has the advantage that it permits one to use the search methods that utilize more than the marginal probabilities of failure of the next members. The latter method has been suggested for somewhat more general structural problems (e.g., *Moses and Stahl, 1978)*.

These methods have proven themselves computationally fast and efficient; it is possible, as was demonstrated in Section 4.1, to carry out reliability assessments of large-scale offshore platforms with these methods. As mentioned, they suffer from several approximations in the event specification and in the computation whose implications are not fully known but are believed to be not serious. The mechanical model is, however, a severe limitation; although it is used in practice, it is not state-of-the-art non-linear, static push-over analysis. The single-failure-force (e.g., axial) and the binary, semi-brittle post-failure model are too simplistic for many practicing engineers. The methods are subject, however, to generalizations. Several authors have included approximate (elasto-plastic) axial-force-moment interaction treatments (e.g., *Thoft-Christiansen and Murotsu, 1986)*. The semi-brittle model is a more rigid constraint. The key characteristic of this method is, however, not the binary member-state, but the finite number of sequences and the finite number of state changes within a sequence. It appears therefore that one need not in principle limit the number of member states to two. Multi-state members will imply some increase in the computation because of the increase in the size of the trees. But the benefits that might be derived from multi-state members are a marked improvement of the mechanical modeling of the post-failure behavior of the members.

6.2 Continuous Systems.

In many circumstances the approximate mechanical and/or loading models discussed in Section 6.1 may be considered inadequate. It appears not to be possible then to use the efficiency of the discrete system (union-of-intersections-of-functions-of-random variables) format. For example, as soon as the force deformation diagram of a single-member is assigned a smooth curvilinear function, it no longer has a finite number of states; hence, the system does not have a finite number of states and one cannot identify the tree-like structure that characterize discrete systems. In still other situations, even though the discrete models above may hold, one may find that computation methods other than those in Section 6.1 become preferable, for example, when the number of member states becomes extremely large. There exist a wide variety of alternative computation schemes, many of which are quite general. The major difficulty with all of them is one of computational cost. We present next two broad classes of these more general methods, together with variations designed to make them more cost-effective.

Simulation Methods. The most straightforward of such methods is the *Monte Carlo* technique, which simply fixes all the random variables at a set of simulated values and then conducts a conventional analysis of the resulting (deterministic) structural system problem. The results will include realizations of member failure events (or not), sequences of failures, etc., (if indeed such discrete representations are meaningful for the system). Note that a realization of the load path is also implied; there is no restriction on how complex it might be. The simple conclusion of this analysis is either that the system has failed or that it has not failed, i.e., the outcome is a system success or system failure.

The probability of failure is estimated as the observed fraction of failures in a set of experiments in which this simulation of variable values and structural system calculations are repeated many times.

There are various computational improvements to the method that increase to some degree its efficiency, e.g., they provide the same degree of confidence in the final answer with a smaller number of samples. The common characteristic of most of these more effective, so-called variance reduction methods is that one must have (from some exogenous source of information) a better idea of where to "look for" the failure event in the space of all random variables. A primary example is the so-called *importance sampling*. These methods are discussed in detail in the companion report by Karamchandani *(1987)*.

The negative side of these methods is recognized when we realize that typical failure probabilities of interest in structural reliability are 10^{-3} or less. Numbers this low may require 10^4 samples (including 10^4 structural analyses) in order to obtain a satisfactory degree of confidence in the final answer. Practically, the variance reduction improvements can at best reduce this number by a factor of perhaps 10. It should be pointed out that in many problems it may be that the structure behaves well within the elastic range for many of the thousands of samples involved. (The variance reduction techniques, on the other hand, will produce a large proportion of non-linear runs, negating in total cost some of their apparent savings.) These simulation methods have the benefits of being simple and direct and being easily used in companion with existing structural and loading analysis programs of the familiar deterministic type.

Reduced Space Methods. We discuss next several rather general methods that are under current investigation by a number of researchers in the structural systems reliability area. A common characteristic of these methods is the reduction of the space of multiple random variables $\mathbf{X}$ to a reduced set. Let us consider a subset $\mathbf{X}_1$ and the remaining set $\mathbf{X}_2$. For specificity in what follows, we shall assume that $\mathbf{X}_1$ is in fact simply a scalar variable, X_1, and more particularly that it is a load intensity variable. In effective applications this might be the most important or dominant structural loading, e.g., the total base shear due to the extreme wave. The reader can easily generalize the discussion below to the situation in which $\mathbf{X}_1$ is a (typically small) vector, a subset of the total vector $\mathbf{X}$. In what follows, we will simply sketch the general patterns of several of these methods; more details can be found in Chapter 2 of the companion document. The general patterns are interesting and should help the reader to recognize the flexibility of methods that are under consideration and help him, when reading specific papers by other researchers, to identify the techniq... used as perhaps falling into one of these general categories.

In discussing these general methods it is useful to think of the exist... a system *g-function*, $g(\mathbf{X})$, of the vector of random variables in the problem. The function need not be uniquely defined; its only necessary characteristics are that it takes on the value zero at the interface between the safe domain and unsafe domain in the multi-dimensional space of the $\mathbf{X}$ vector, negative values in the unsafe domain, and positive values in the safe domain. For a member the safety margin, $R-S$, is such a function (of $X_1=R$, capacity, and $X_2=S$, load effect, for example.) For complex system reliability problems the function is primarily only of conceptual value. Typically, it can only be implicitly identified because it normally takes a (deterministic) computer algorithm to evaluate system failure or not-failure given a set of values of $\mathbf{X}$.

In the systems discussed in the previous sections, for example, $g(\mathbf{X})$ would represent the union of intersections of the member-failure-events. In this case one can visualize $g(\mathbf{X})=0$ as an envelope of the union a set of intersecting curves, Fig. 6.1.

In the **first method**, which we will call simply Method A, the first step is to **fix** the value of X_1 at one of a small set of pre-selected values x_1^i, i=1, 2, ...N_s, in which N_s is the number of values to be selected. Step 2: a Monte Carlo analysis is run on the $\mathbf{X}_2$ vector, selecting N_i sets of values from the *conditional* distribution of $\mathbf{X}_2$ given X_1 (in many cases X_1 may be chosen so that $\mathbf{X}_2$ is in fact independent of X_1, simplifying this distribution). Step 3: with the selected X_1 value and for each of the sets of sample $\mathbf{X}_2$ values, one conducts a conventional structural analysis to determine whether the structure fails or does not fail. The result after N_i sets of $\mathbf{X}_2$ values is an estimate of the probability of failure of the system for that selected value $X_1=x_1^i$. Repetition over i=1, 2,...N_s yields points on the function $P_f(x_1^i)$, i.e., the probability of failure on the system conditional on $X_1=x_1^i$. (In some reliability work this function is referred to as a *system fragility function*, if indeed X_1 is a dominant loading variable; this is, for example, the case in the nuclear power plant seismic reliability analysis community.) Step 4: one calculates a net system failure probability by integrating this last function weighted by the probability distribution (density or discretized mass function representation) of X_1 over all x_1 values.

This method is particularly effective if the probability distribution of X_1 has a relatively large variance, implying that its probability density function (PDF) is relatively slowly varying over the range of x_1 values in which the conditional system failure probability, $P_f(x_1)$, passes from near zero to near one. This flatness of the PDF has been found to be the case under loadings such as the seismic load where the coefficient of variation on the peak ground acceleration may exceed 100 percent. In some offshore cases we may expect the PDF of wave load effect (if not maximum wave height) to have a relatively large variance; this was observed in the jacket illustration in Section 4.1. In extreme cases, such as the seismic case, the system fragility function can, to a first approximation, even be replaced by a step function from zero to one at the value of x_1 corresponding to 50 percent level of $P_f(x_1)$. In this case the system failure probability is approximately equal to the probability that load variable exceeds that median x_1 value. A distinguishing characteristic of Method A is that one must conduct several Monte Carlo analyses, one at each of the values of x_1^i. Appropriate sample sizes N_i change as a function of where one is on the fragility curve. Importantly, they may be very small (on the order of 10) near the median, which may be the most critical area if, as discussed above, the variance of X_1 is relatively large. As one passes toward the tails of the system fragility curve one would like larger sample sizes to obtain the same standard error of estimation; one may often tolerate, however, larger standard errors of estimation in these tails and hence not such large increases in sample size as one might anticipate. To take advantage of these potential savings (over, say, simple Monte Carlo), one must choose X_1 well, and one must know in advance or by clever sequential searches just where the median and tails of this fragility curve lie, i.e., what values x_1^i to choose. Structural and reliability experience can be a prime source of such information.

The **second method**, Method B, uses the same X_1 and $\mathbf{X}_2$ sub-vector approach discussed in Method A. It has the following steps: Step 1: one simulates from the distribution of the sub-vector $\mathbf{X}_2$ a set of values $\mathbf{x}_2^k$, for k=1, 2,...N, in which N is the Monte Carlo sample size. Step 2: for each set one conducts a search over the X_1 axis to find the value x_1^{k*} where the system is just on the verge of failure. At this point we would say that the system g-function, $g(\mathbf{X})$, equals zero. Step 3: the probability of failure is calculated *conditional* on the specific selected set of $\mathbf{x}_2$ values, i.e., $\mathbf{x}_2^k$. For the case in which X_1 is a load random variable, this can normally be done by evaluating the probability that X_1

takes on a value larger than x_1^{k*}. In words, this probability is the ***conditional*** complementary cumulative distribution function of X_1 given $\mathbf{X}_2$. (We noticed in passing that if X_1 has indeed been selected to be independent of $\mathbf{X}_2$ vector, this is a simple calculation.) Let us call this value P_{f_k}, k=1, 2,...N. Step 4: one can now estimate the system failure probability by simply summing the conditional probabilities of failure just identified and dividing the sum by N.

Some particular applications of this method *(Esteva, 1985; Bjerager, 1986)* have taken advantage of the fact that the $\mathbf{X}$ vector can be transformed into a polar coordinate representation, i.e., in terms of vector length, R, and a set of direction angles or direction cosines. The length R plays the role of X_1. If, further, the $\mathbf{X}$ vector has first been transformed into $\mathbf{U}$, a normalized independent Gaussian vector, then the direction cosine vector (playing the role of $\mathbf{X}_2$ in the description above) is uniformly distributed on the unit sphere, simplifying its simulation, and the length R is independent chi-distributed (with $n-1$ degrees of freedom, n being the dimension of vector $\mathbf{X}$), simplifying the calculation of the probability of failure given the value(s) associated with the (transformed) g-function being zero. The price paid for the simplicity on the so-called ***directional search method*** is the enhanced difficulty of carrying out the search in the transformed space.

The basic characteristic of Method B is that, relative to simple Monte Carlo, one gains a much greater amount of information with each realization in the simulation because one searches for x_1^{k*} and obtains a failure probability P_{f_k} for each realization (as opposed to obtaining nothing but a 0 - 1 (binary) observation as in simple Monte Carlo). Given more information per realization it is not surprising that one can reduce the size of the sample, vis-a-vis simple Monte Carlo, necessary to gain comparable confidence in the estimate (e.g., 95% confidence bands of the same percentage in width). The price is paid, however, in the search process; the structure will have to be analyzed for several to many different values of x_1^k at each Monte Carlo realization in order to identify (at least approximately) the edge of failure surface, i.e., the value of x_1^k such that the g-function equals zero. In some cases this can be done by a direct incremental load application, implying in effect only a single non-linear analysis.

The **third method**, which we will call Method C utilizes a mixture of techniques. Step 1: select set of values from the entire vector $\mathbf{X}$. (These values may be selected by simulation or by what is referred to as ***experimental design***. Step 2: for each selected set of values, $\mathbf{x}^i$, i=1, 2,...n, calculate the value of the system g-function, $g(\mathbf{X})$. Step 3: using a technique of linear or non-linear regression analysis, approximate the g-function by a simple polynomial obtaining an explicit (approximate) function $\tilde{g}(\mathbf{X})$. Step 4: calculate the probability of failure by conventional functions of random variables techniques as $P_f = P[\tilde{g}(\mathbf{X})<0]$, using, for example, one of the modern methods such as FORM or SORM. The effectiveness of Method C is not well known as yet; one is concerned, for example, that in general the precise numerical value of the g-function has no meaning except for values g=0 (e.g., any monotonically increasing function of a g-function is also a valid g-function), and what this may imply about the resulting effectiveness of the approximation. In any case one should use some form of penalized objective function that favors a good fit near g=0, and, if efficient to implement, a better fit for higher values of $f_{\mathbf{x}}$, i.e., near the most likely failure point. This method tries to extract more information from each Monte Carlo sample by using the value of the g-function as a distance measure (however imperfect it may be).

The **fourth method**, Method D, involves more steps. Step 1: select a set of values $\mathbf{x}_2^k$ of the sub-vector $\mathbf{X}_2$, i=1, 2,...N_2. Although not necessary, it has been found desirable and successful *(Veneziano et al 1983)* to split this sub-vector in turn into a vector of "more important" variables $\mathbf{X}_{2ss}$ and "less important" variables $\mathbf{X}_{2rs}$. The first set, $\mathbf{x}_{2ss}^i$ may be selected by some form of experimental design or simply simulated; the second set, $\mathbf{x}_{2rs}^i$ should be simulated from its *conditional* distribution given the first set. As in the second step of Method B, search along the X_1 axis to find that value x_1^{i*} such that the system g-function is (approximately) zero. (This is the expensive, possibly multiple structural analysis step.) Step 3: by regression analysis (sometimes called *response surface* in this context) fit a function of X_1^* to sub-vector $\mathbf{X}_2$ (or more specifically to the sub-vector $\mathbf{X}_{2ss}$) in a convenient, explicit polynomial form $X_1=h(\mathbf{X}_2)$. Notice that the error term in the regression analysis captures not only the "modeling" error (the difference between the exact and the fitted value of X_1^* versus $\mathbf{X}_2$), but also any error associated with the randomized values of $\mathbf{X}_{2rs}$. Step 4: calculate the probability of failure by a function of random variables method such as FORM or SORM, using the approximate analytical g-function $\tilde{g}(\mathbf{X})=h(\mathbf{X}_2)-X_1$. The random error term, ε, from the regression analysis, with its zero mean and presumed Gaussian distribution, can also be absorbed at this step in an expanded g-function, i.e., by increasing the vector $\mathbf{X}_2$ to include ε.

The potential effectiveness of the method is evidenced by the fact that it has been used by its authors *(Veneziano et al, 1983)* to calculate the probability of failure of a multi-story, multi-bay reinforced concrete structural frame with complex non-linear, cumulative-hysteretic failure criteria under severe transient dynamic stochastic earthquake ground motions. The method appears to be both flexible and efficient. It is flexible in that the engineer can categorize his random variables into the three categories indicated X_1, $\mathbf{X}_{2ss}$, and $\mathbf{X}_{2rs}$ depending on their relative variability and importance in the problem. The method is efficient in that it takes advantage of the relative strengths of regression analysis (Step 3), FORM-like methods (Step 4) and Monte Carlo and or experimental design) (Step 1). It does, of course, require in Step 2 the expensive process of searching for the failure surface, i.e., for g=0, by, in general, multiple structural analyses. This must be done N_2 times. The objective of the fitting and smoothing process is, of course, to minimize this effort by keeping the size of the set selected in Step 1 as small as possible.

The exposition of these methods has intentionally been somewhat detailed here in order to give the reader a flavor of the possibility of combining various techniques of probabilistic analysis in order to efficiently conduct structural systems reliability calculations on large general systems. One is also left with the impression that researchers will continue to invent variations and permutations on these and similar ideas, and that it will be a difficult research task to evaluate these various strategies for any but a specific example problem or sample of example problems. This situation is somewhat analogous to that found in the field of mathematical programming or optimization, including even *linear* programming, whose many available algorithms are in fact compared empirically on sets of sample problems for their relative efficiency. In this age of expert systems one can envision a computer program that would guide the structural engineering user through a "cookbook" of such methods using the experience of individuals with a variety of problem experiences. The information that would be useful to help select among the methods would include not only exogenous information such as relative variability of the different random variables and, equally importantly, good mechanical understanding of the structure's behavior (particularly in "partially failed" states), but also problem-specific internally generated information. The latter would include sensitivity coefficients or partial derivatives of failure probabilities with respect to variations in the parameters. Recent developments in FORM and SORM include the ability to calculate such coefficients *(Madsen et al, 1986)*. The selection of methods, sample sizes, and other parameters can even be sequential, changing as information becomes available during the problem solution.

6.3 Additional Topics

In this section we will collect a number of issues that are of major importance in the realistic analysis of the reliability of large offshore structural systems. None as yet, however, has received a great deal of research or application. Therefore, our treatment can be brief.

Loads: It is not unexpected that the topic of characterization of environmental loads for the use in structural systems reliability calculations has not yet been given systematic treatment. It is only within the last decade that either systems reliability or structural loads analysis has individually been given significant attention. It is natural that the first work in structural systems reliability would focus on single or simple sets of time invariant loadings. It is equally natural that the study of multiple random load processes would focus initially on individual members. In this section we will do little more than point out some of the necessary problems for future identification and solution, problems related to loads on structural systems.

It is helpful to consider in parallel a set of increasingly more complicated, realistic representations of loads on a (scalar-capacity) member with the corresponding cases in a system. The simplest member-level problem is that of a load represented by a scalar random variable; the parallel problem for a system is a spatial pattern of loads scaled by a single random variable, as we have discussed in previous chapters. We have not pointed out, however, that there is no difficulty in principle in treating the case when the spatial load pattern changes systematically (functionally) with the value of the scaling variable. A practical case is one in which the force pattern changes with increasing wave height. The next problem in difficulty involves, on one hand, a pair or vector of simultaneous scalar random variable loads (effects) all applied to the same member cross-section.) The problem appears to reduce to simply considering the sum of those loads, which is in turn a scalar random variable. Given that some of the loads' effects may have opposite signs, there is implicit in this solution the assumption that the loads are proportionally increased. The parallel problem in a system is already considerably more complex, for, as we have discussed above in Section 6.1, one should be most explicit about specifying the path or trace (relative rates) by which the various loads increase to their final (random) values. One can already then not escape at least this limited consideration of these loads as time-varying process. To reduce them, as is often done in both in deterministic and stochastic practice, to a simpler problem by assuming that they increase proportionately from zero is to gain the analysis benefits that one can derive from that representation; it is not necessarily to obtain a realistic representation of the loads on a structure *(Bjerager, 1984; Wen and Chen, 1986; Karamchandani, 1987).*

For a member the next most difficult problem is to consider the (single, for now) loading as a general (non-monotonic) function of time. Consider the member reliability problem. If the response of the structural system in which it resides can be assumed to be static, then the analysis of this load random process reduces to finding the characteristics, i.e., the probability distribution, of the random variable defined as the maximum of this load process over the lifetime of the structure. If the structural response is dynamic, one must first conduct a random vibration analysis and then solve it for the distribution of the maximum load effect in the member. Either of these maximum random variables can be used together with the probability distribution of the capacity to find the probability of member failure. The parallel problem for structural systems safety analysis is analogous provided there is no damage, i.e., no change of the system out of the linear-elastic intact state. However, even in the simple static load process case, one can envision much more complex behavior in the system. For example, as the load is applied and removed or reversed and applied again, as it might be during a sequence of severe waves in a large and intense storm, or in a sequence of intense storms, one must consider the structural system passing into partially damaged states. Some members may yield, for example, followed later by unloading and strain reversal in these "failed" members. Deterministically, we are dealing with problems which include topics such as shake-down and ratcheting. Probabilistically, we must be prepared to follow the element states and system states in time as the load increases and decreases. The analogous computations are made commonly in the industry today during non-linear dynamic analyses of large jacket structures located in severe seismic areas, but in a deterministic context. A clear implication is that as we pass to probabilistic structural systems we must keep much more information about the character of the loading process, and transmit from the loads analyst to the structural analyst, much more information than that retained in the simpler member analysis problem.

Finally, if one is looking at the problem of multiple random load processes and a single member behaving linearly until its failure, we can analyze the load processes for the probability distribution of the maximum of the load effect, i.e., a linear combination of such load processes. This is the so-called "load combination problem" that has drawn the attention of a number of research and code-development engineers in the last fifteen years (e.g., *Wen, 1977; Larrabee and Cornell, 1979; Borges and Castenheta, 1972)*. Although not a trivial problem, at least two rather robust methods of analyses are available ("load coincidence" and "point crossing"). For the structural systems reliability problem, the analysis under a combination of several loads processes is non-trivial, but in principle little more difficult than we were forced to face in the case of the problem of loads represented nominally as simply a pair of random variables, as discussed above. As we saw in Section 6.1, even the simple discrete-system analysis involves the treatment of the out-crossings of random load processes through component-failure segments and following the successive partial-damage states of the system. We can anticipate that the practical reliability analysis of offshore members or structures for time-varying loads will involve a variation on the long-term/short-term analysis of environmental events. Specifically one can envision a method in which random events occur and each event is associated with simply a vector describing the major characteristics of the loading event, e.g., the significant wave height, the dominant wave direction, average current velocities, etc., during the (perhaps storm) event. The structural reliability analysis can then be conducted within that event as if the loads could be represented by a vector of random variables, as discussed in Section 6.1. Proper account must be taken of the load path, as was discussed there, and potentially of dynamic effects superimposed upon the basic intensity random variables for the event. Such an analysis procedure would also be relatively straightforward in principle for structural systems provided one ignores the possibility of damage in one event being "carried over" to the next event. If this possibility cannot be ignored, i.e., if there is effectively a "system-level cumulative damage," then more complex methods must be adopted. The first step in this direction is to assume the so-called Markov property, i.e., that one can characterize the state of the system adequately by a vector of random variables which one varies from event to event. This method has been demonstrated by Veneziano *(1981)* for a simple non-linear oscillator undergoing progressive collapse and recently by Wen and Chen *(1986)* for elementary structural systems.

Practically speaking, for the foreseeable future, because we are pushing not only the state-of-the-art of systems reliability and stochastic processes, but also often the state-of-the art of structural mechanical modeling and analysis, including non-linear dynamic behavior of large structures, we must anticipate that the solutions are going to be limited to those that involve a great deal of simulation. Of those methods discussed in Section 6.2 the pure Monte Carlo simulation and Method D lend themselves to the analysis of these general system loading problems. (In the latter case short-term time variation can be treated with the vector $\mathbf{X}_{2_{rs}}$.)

Fatigue. The stochastic analysis of structural systems undergoing member fatigue has very recently begun to be considered by researchers. The elementary, ideal system of the type discussed in Chapter 3, has been analyzed by Stahl and Geyer *(1984)*. More recently Rackwitz and his co-workers have began to address the more general but still discrete structural system of the type discussed in Section 6.1 for fatigue (*Guers and Rackwitz, 1986*). Interesting systems analysis questions here concern whether there is significant damage process correlation induced in the member failures by the fact that they are subjected to common loadings. Stahl and Geyer *(1984)* suggest that the correlation between times to failure of the members in the ideal system may be as low as 32%. Further one is interested in the systems aspects associated with the failure of one member increasing the stress level in the remaining members and hence accelerating their damage accumulation processes, but this is not unlike the basic overload problem we have been discussing throughout this chapter. A final question of interest is whether the gradually accumulating fatigue damage may interact significantly with the capacities of the members with respect to occasional overloads. As discussed in Chapter 2, Guers and Rackwitz *(1986)* have studied this problem and concluded that, at the individual member level at least, the interaction can be ignored in terms of its impact on reliability; this fatigue study was based on linear elastic fracture mechanics crack growth. It is this conclusion that permitted us to separate the fatigue problem in Chapter 2 from the extreme value or overload problem, and therefore to conveniently consider fatigue as if it were an exogenous damage condition.

Uncertainty Analysis: To the author's knowledge, outside of the above mentioned nuclear-power-plants-seismic-safety-analysis convention of separately propagating through the systems analysis the effects of uncertainty (not only individual parameter values, e.g., means, standard deviations, etc., but also in the models, e.g., shapes of distributions and mechanical modeling assumptions), there has been no widespread application of uncertainty analyses in the structural systems area. It does appear to be a problem which can be treated by the same general techniques one would use for uncertainty analysis in member-level reliability analysis. Therefore we need not discuss it in detail here. In brief and in simple form this involves repeating the reliability analysis for each and every possible combination of the possible values of the uncertain parameters and/or models, and assigning to each of these sets of values a probability (or "degree-of-belief") value. The net result is an "uncertainty distribution" on the probability of system failure. These techniques are described in some detail in the nuclear industry's PRA Procedures Guide *(1983)*. Certain simple systems analysis problems, e.g, those involving unions of intersections of independent binary events with specified probabilities (as opposed to events defined by the value of a g-function of the vector **X**) can be treated at least approximately by more analytical methods.

Full-Scope Systems Analyses. These are analyses involving, and often emphasizing more, questions associated with the operation of the general oil production system, as opposed to the structure supporting its platform. The analyses should include the spectrum of possible accidents associated with the process itself as well as exogenous accidents such as ship collisions. They should also include the interaction of possible operational accidents with structures, for example, explosions. The impact of structural-mechanical failures in turn upon the accident sequence may also be a factor. For example, an explosion may cause partial structural collapse of the deck which in turn may induce the loss of fire fighting capabilities and/or personnel escape access. It is the author's understanding that in the several (proprietary) PRAs of offshore production facilities that have been conducted in Norway and England these kinds of interactions have been dealt with explicitly. Indeed a primary purpose of these studies has often been to address the layout of the facilities on the platform relative to each other and relative to structural elements. The depth of treatment of the structural system within these larger full-scope reliability analyses has, to date, been relatively limited, in part because the budget level implies that the systems analysis team must allocate its resources according to the perceived relative importance of the accident types and system components. Also, the details of the structure, its layout and member sizes, may not be well enough resolved at the early conceptual stage of the design process to permit more than cursory structural system reliability evaluation. Finally, it may well be that the level of structural systems analysis that has been implied by discussions in this report are more detailed and resource-intensive than is appropriate given the importance of the contribution of this part of the problem to the total system.

The techniques for the full-scope system analysis have been developed in many industries, initially apparently in the aerospace industry, reaching fuller development in the nuclear industry in 1970's and early 80's, and being adopted and extended by other industries such as the offshore industry and more recently the chemical industry in the last ten years. Perhaps the most effective single reference known to the author is the nuclear-industry-sponsored PRA Procedures Guide *(1983)*. It should be remembered, however, that the scope of the studies envisioned by this guide is indicated by the typical two-million-dollar budget (perhaps an order of magnitude greater than that spent on current offshore full-scope risk assessments.) The report includes a full discussion of the event-tree and fault-tree methods (including their superposition), as well as treatment of uncertainty in parameters and models.

There are two chapters in the Guide on what are called ***external events***, such as seismic events, tornados and fire. The readers of this document will find these chapters particularly interesting with respect to their treatment of the behavior of (passive) structures and (passive and active) electro-mechanical systems under extreme environmental loadings. The basic method of analysis is somewhat like that discussed in Method A of Section 6.2, although in place of Monte Carlo analysis the system failure probability is calculated by event-trees and fault-trees at each of a small set of values of the environmental load intensity, e.g., the peak ground acceleration level of the earthquake motion. This is followed by an integration of this conditional system failure probability (i.e., the system fragility curve) multiplied by the (discretized) probability density function of the lifetime maximum earthquake level (referred to as the ***seismic hazard curve***), and integrated. There the systems analyses are typically simplified to assume either independent or perfectly dependent component failure events conditionally upon the intensity level of the earthquake. This greatly simplifies the probability calculations. The probability of failure of each mechanical or structural component in the system at a given peak ground acceleration level is read from an input *component fragility curve*. The component fragility curve is effectively the cumulative probability distribution of the capacity of that member with respect to the external load, i.e, the peak ground acceleration. The development of each of these individual member fragility curves may be rather complex because it must consider the mechanical effects of the soil and the structure, their (non-linear) dynamic responses to the earthquake and, for within-structure equipment, their effect upon the base-motions felt by those pieces of equipment. In addition, these curves depend on the estimated structure or equipment capacity with respect to these motions (e.g., *Cornell and Newmark, 1978; Kennedy et al, 1980)*. Because it is required, in addition, to conduct an uncertainty analysis, it is necessary also to specify uncertainty bands about these member-fragility curves and uncertainty bands about the seismic hazard curve. The system fragility curve uncertainty bands and final system failure probability (e.g., core melt probability) are deduced from the analysis by the methods discussed just above.

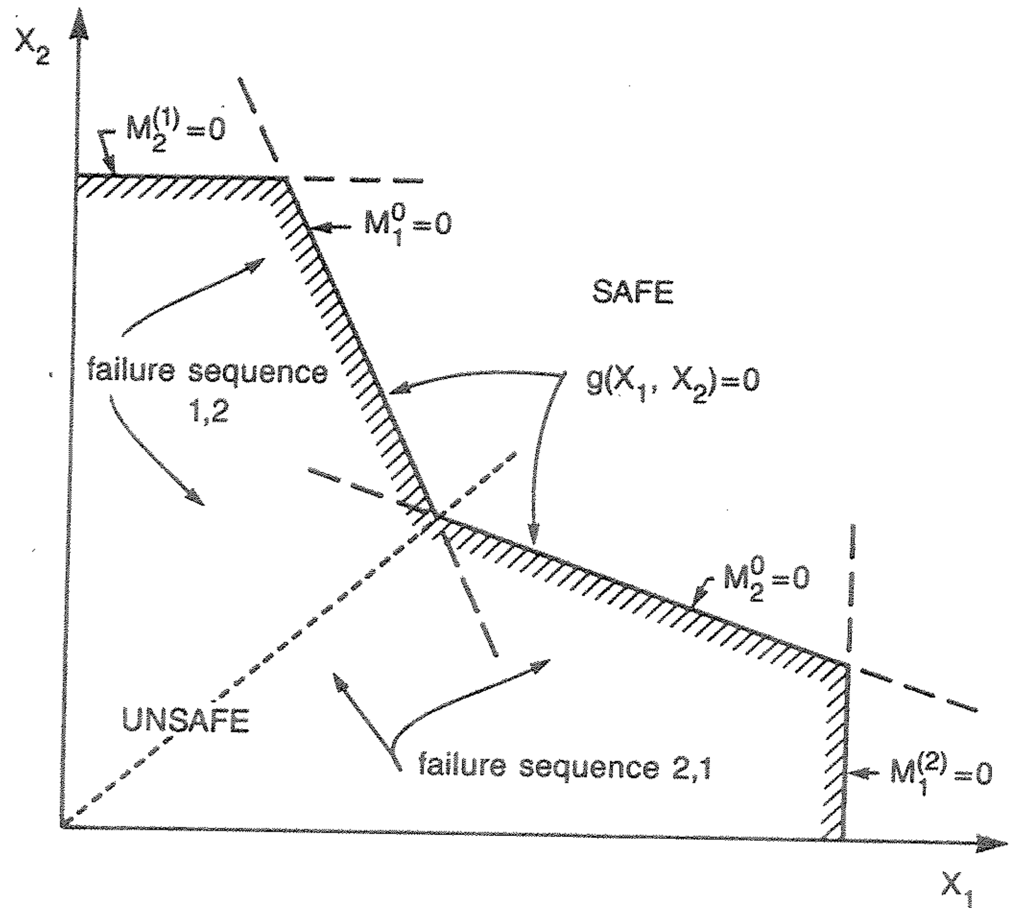

Fig. 6.1 System-Level g-function Showing Member-Level g-functions at Different System States.

Chapter 7

CONCLUSIONS AND SUGGESTED DEVELOPMENT AND IMPLEMENTATION

We have reviewed and critically evaluated the recent rapid developments in structural systems reliability research and its application, all from the perspective of the needs of the offshore industry. Based on this review and supported by the experience obtained in our case study of the Lloyd-Clawson jacket platform (Chapter 4), we have reached several conclusions.

There is a gap to be filled. On one hand, there is rapid widespread industry development in reliability-based description and specification of loads, materials, and member behavior. On the other hand, there is a need to address quantitatively questions such as redundancy, residual strength of damaged systems, robustness, relative safety of novel platform concepts, economic inspection strategies, treatment of uncertainty in new and existing platforms, etc. These are all questions beyond the reach of member-based reliability analysis. Structural systems reliability analysis is the tool necessary to fill this gap.

With the rapid recent research developments in structural systems reliability analysis, it is now possible to conduct such an analysis efficiently and routinely on typical offshore structures, at least approximately and under simple static loadings. We made just such analyses in the Statoil/Stanford study of the Lloyd-Clawson platform. With these new methods, it was possible to do much more even than simply make a single systems reliability assessment of a given structure. Systems reliability analysis was used repeatedly to investigate the reliability implications of alternative configurations (e.g., X versus K bracing), alternative horizontal bracing sizing, alternative load parameters and member post-failure behavior models, the loss of various critical members, etc. The current procedures are efficient enough to permit systems reliability to be used as a tool by knowledgeable offshore structural engineers without extensive prior experience in systems reliability theory. In short, systems analysis methodology is both accessible enough and efficient enough to be used effectively in advanced industry practice today.

As currently implemented, however, these efficient systems analyses methods are too elementary in their mechanical modeling assumptions to meet all the expectations of an advanced technology such as the offshore structures community. The cause is a fundamental one. The methods are very efficient precisely because their member-behavior models are so simple. The most popular of these models is the simple elasto-plastic model. Some methods include more general but still only two-state (binary) member behavior: an intact state in which the member is linear elastic and a failed state in which no stiffness remains. (See Fig. 3.5.) If the elasto-plastic version of this two-state member is accepted, then a broad selection of available systems analyses techniques is available. The most advanced is apparently the offshore-oriented system under development by Veritas Research. With the two-state ductile or semi-brittle member model, one can use familiar linear structural analysis procedures which are fast and familiar. Several reliability techniques are available to exploit this characteristic. The more prominent are the Member Replacement Method and the Incremental Load Method. (Chapter 6.1)

These methods can, however, be substantially improved with respect to their mechanical and load modeling assumptions without significant loss in the efficiency that makes them so practical today. This enhanced method would be, we believe, a strong effective compromise between the current capabilities of the systems reliability community and the current industry practice with respect to non-linear structural analysis. It would be capable of systems reliability analysis of three-dimensional, framed structures and foundations under an extreme static wave load.

On the other hand, if we turn to the capabilities of systems reliability analysis with respect to the more advanced models of offshore structures that are needed today, including continuous structures such as TLP hulls, load reversals, dynamics, fatigue, etc., we must conclude that their potential is currently limited to small-scale, special-study applications. Several of these methods, e.g., Monte Carlo, etc. (Chapter 6.2), may be general enough but they are impractical in terms of the computation time required for the analysis of highly reliable large scale structures with complex loading and behavior. To study such problems today, one must be a specialist and exploit special simplifications in the model, the analysis, and/or particular reliability tools. There appears to be no efficient general purpose program that can interface effectively with the spectrum of existing industry loading and structural analysis packages, as would be desirable if broad use of systems reliability for such problems is to be effected.

Finally, we identified several particular important topics where the systems reliability analysis research community has yet to make virtually any progress to date. The first of these is in the area of representation of random structural loads for use in systems reliability studies. It is very clear that the history here has been based strongly on the consideration of individual structural member reliability. In passing, as the community is now, toward structural systems problems, a new set of issues arises. These include proper treatment of the load path (a topic which is often not treated realistically even in deterministic non-linear structural analysis), multiple simultaneous loadings, and sequences of sets of loadings. In all of these cases, systems reliability seems to introduce problems that had not been encountered previously in member reliability. Secondly, the separate and careful treatment of uncertainty, as distinct from randomness, has yet to be introduced explicitly in structural systems reliability analysis. The exception is in the application of such methods to seismic analysis of nuclear power plants. The methods, however, have not been developed for the many additional needs particular to the offshore industry, e.g., fatigue questions, inspection questions, new versus old structures, information sensitive codes, etc.

Recommendations. Based on these many observations, we would like to make several suggestions for a strategy to extend further the effective implementation of structural systems reliability in the offshore industry. These include (1) developing mechanically-enhanced, efficient methods for certain classes of structures for near-term use, (2) the continuation of various studies ranging from special projects through generic topics to policy issues, and (3) general methodological developments.

It appears that it will be feasible in the short-term to develop a stand-alone computer program capable of exploiting the fast, efficient linear structural analyses procedures and the Member Replacement Method of structural systems reliability. In this form, the method would be useful immediately in practice. It would permit a large number of industry engineers to gain experience with systems reliability on a large sample of structures, providing the industry with calibration points with respect to the reliability, redundancy, and robustness of current jacket structures. The necessary improvements include both the mechanical modeling assumptions and the loading assumptions.

We recommend encouraging the current expansion of activity in the use of structural systems reliability through various special studies, usually conducted by a specialist in structural systems reliability and an engineer closely familiar with offshore engineering practice. Specific projects that are underway or that can be envisioned include different individual types of novel, compliant structures, the application to continuous hulls by finite-element analysis, full-scope (layout/operation/inspection/structure) studies, etc. Other such special studies might be aimed at particular generic topics such as the redundancy issue, seismic questions, fatigue questions, foundations, code factors, and particular load types. It will also be useful for the industry to continue to pursue various policy/systems reliability studies such as those associated with the geriatrics problem, damaged structures, or novel systems. Questions of interest include what level of systems analysis is necessary, what kinds of study should be conducted under such situations, which probabilistic model developments are most needed, etc.

Finally, a number of general methodological developments appear to be necessary. One should be a "general solver" capable of being coupled with the existing load analysis and non-linear and/or dynamic structural analysis programs used in the industry. Such a solver should be virtually independent of the nature of the structural problem being addressed. Secondly, as mentioned above, the treatment of uncertainty, including its specification and its updating, appears to be a common thread among many of the industry needs, as well as an area of very incomplete development in research. Finally, the particular methodological developments necessary to the offshore industry would appear to focus on systems reliability procedures for large "one-off" structures using relatively advanced mechanical methods (as opposed to the more repetitious, routinely designed structural situations involved in the building and bridge industry), particular loading types (e.g., waves and ice), and important dynamics questions (specifically detailed, non-linear dynamic analysis of deep-water systems).

Appendix A

COMPLETE STRUCTURAL SYSTEMS RELIABILITY FORMULATION

$$p_{f_{SYS}} \approx p_{f_{OL}} + p_{f_{EX}}$$

$$\approx p_{DL} \cdot L \cdot O \cdot D_0 \cdot C_0 \cdot R_0 \cdot M_S$$

$$+ \sum_i \sum_j p_{ij}\, L \begin{cases} 1 \cdots \mathit{if\ system\ cannot\ survive\ without\ member}\ j \\ (p_{f_{damaged,ij}} + p_{f_{repair,ij}}) \cdots \mathit{otherwise} \end{cases} \tag{A.1}$$

$$p_{f_{damaged,ij}} = (1-I_{ij}^{Q})\, p_{DL} \cdot O \cdot \overline{T}_j \cdot \overline{D}_j \cdot \overline{C}_j \cdot \overline{R}_j \cdot M_S \tag{A.2}$$

and

$$p_{f_{repair,ij}} = \widetilde{I}_{ij_{net}}^{Q} \cdot p_{DL} \cdot O \cdot D_0 \cdot \widetilde{C}_j \cdot \widetilde{R}_j \cdot L_{R_{ij}} \cdot M_S \tag{A.3}$$

in which $\overline{T}_j$, the expected time spent damaged (before repair) is the lesser of $L/2$ and

$$\overline{T}_j = \frac{T}{2} I_{oj}^{Q} + \frac{3T}{2}(1-I_{oj}^{Q}) I_{oj,2}^{Q} + \frac{5T}{2}(1-I_{oj}^{Q})(1-I_{oj,2}^{Q}) I_{oj,3}^{Q} + \cdots \tag{A.4}$$

and in which $\widetilde{I}_{ij_{net}}^{Q}$ is the probability of damage detection prior to failing in the damaged state, which is approximately the probability of *ever* detecting the damage or

$$\widetilde{I}_{ij_{net}}^{Q} = I_{ij}^{Q} + (1-I_{ij}^{Q}) I_{oj} + (1-I_{ij}^{Q})(1-I_{oj}^{Q}) I_{oj,2}^{Q} + \cdots \tag{A.5}$$

Definition of Terms and Comments

$p_{f_{SYS}}$ Probability system fails during economic lifetime.

$p_{f_{OL}}$ Probability system fails due to overload in lifetime.

$p_{f_{EX}}$ Probability system fails due to "exogenously initiating event", e.g., ship collision, dropped objects, fabrication error, and perhaps even fatigue.

p_{DL} Annual probability of exceeding design wave load, e.g., 10^{-2}.

L Duration of economic life, e.g., 20 years.

O "Overload factor", defined such that $p_{DL} \cdot L \cdot O$ is the (lifetime) failure probability of the "code base" member, i.e., one fully stressed under design load to code limit. This allows for randomness in capacity, combined loads, etc. $p_{DL} \cdot L \cdot O$ corresponds to the (lifetime) failure probability associated with member level design as in the API LRFD project basis, i.e., $\Phi^{-1}(\beta_{code})$. Typical value of O if the (lifetime) β_{code} is 3.0 is 10^{-2}, because $\Phi_{-1}(\beta_{code})$ is 10^{-3}.

D_0 This factor adjusts, if necessary, the base member from one stressed-to-the-code-limit to the most heavily stressed member in the structure; strictly, $p_{DL} \cdot L \cdot O \cdot D_0$ is the largest lifetime failure probability among all individual members (or locations) in the structure. This would be the governing probability if "worst-member" were the basis of the analysis. D_0 may be less than one, if, for example, the designer intentionally chooses to be more conservative than the code. D_0 will be greater than one, perhaps, for older, pre-current code structures, or for operating structures being re-assessed for previously ignored (but now found important) behavior modes or for new loads (e.g., heavier operating deck loads).

C_0 System complexity term. Defined such that $p_{DL} \cdot L \cdot O \cdot D_0 \cdot C_0$ is the lifetime probability of failure of (at least) one member in the system. A crude, conservative, approximation: C_0 = sum of individual members' annual failure probabilities (each under its most critical load case) divided by $p_{DL} \cdot O \cdot D_0$, the annual failure probability of the most-likely-to-fail member or "base" member. Exact C_0 considers correlations (e.g., due to common load), etc. C_0 may vary from 1 to n, the number of members in the structure. p_{DL}, L, O, and D_0 are individual member factors; they do not reflect importance of the member to the system. C_0 and R_0 (to follow) are, in contrast, *system-related* factors. (Formally, C_0 = Probability of the union of all first member failures divided by $p_{DL} \cdot L \cdot O \cdot D_0$).

R_0 System redundancy term (of the intact structure). R_0 is the conditional probability of total system failure given at least one member has failed. Defined such that $p_{DL} \cdot L \cdot O \cdot D_0 \cdot C_0 \cdot R_0$ is the lifetime system probability of failure using standard (pseudo) static pushover models (see M_S) with some simplified failure criterion. R_0 is one for a statically determinate structure, and less for structures with post-first-member-failure capacity. $O \cdot D_0 \cdot C_0 \cdot R_0$ represents the "reserve strength" (see Lloyd and Clawson, 1983) of the system in probabilistic terms. It is defined relative to the code design level. A typical value may be 10^{-2} or less. The product $C_0 \cdot R_0$ (which may be greater or less than 1) represents the net system effect versus the "base" member (which may be the "code member", if D_0=1). Thus $C_0 \cdot R_0$ equals the ***system factor***. (Formally R_0 is the lifetime probability of failure of the system under all loading events, e.g., multiple wave directions, seismic, etc., as determined from appropriate static pushover representations, divided by $p_{DL} \cdot L \cdot O \cdot D_0 \cdot C_0$).

M_S — ***System modeling factor***. It corrects the analysis from the static push-over basis (which is proposed here as the basis of R_0, for simplicity and practicality) to the true case involving repeated load reversals, with perhaps dynamic response. *Ms* recognizes ductility, cumulative deformations, shakedown, more refined failure criteria, etc. This value may be largely judgemental (supported by deterministic dynamic analysis, especially in the seismic case and by simplified special reliability studies) until system reliability calculation capabilities improve to cover all these complexities.

p_{ij} — The annual probability of occurrence of "scenario" i (p_i) times the (conditional) probability of damage/failure of member (or set of members) j, given scenario i, i.e., the probability of the joint event scenario i and damaged member(s) j. Scenarios include different types of exogenous initiators (and different levels of severity, if appropriate, e.g., ship size). The member (set) j collection may include several levels of damage to each specific member (e.g., mildly bent to total rupture). Recent studies suggest that it may not be a bad approximation to include member failure due to fatigue/fracture through this vehicle, even though it is not strictly "exogenous". A poorly constructed member (e.g., an improper weld) can certainly be included. ($p_{ij} \cdot L$ is the lifetime probability of the joint event.) Note if the structure fails immediately given the damage state j, then $p_{ij} \cdot L$ is the probability of system failure due to this "cause".

$pf_{damaged,ij}$ — The probability of subsequent system failure while still in the damaged state j following scenario i, i.e., prior to detection (and assumed instant repair, see $\bar{T}_j$) of damaged member(s) j. This term considers the residual strength of the system and the (less-than-lifetime extreme) sea states that may occur in the time until detection. That time is affected by inspection policies, T and I^Q_{ij}, see below.

$p_{f_{repair,ij}}$ The probability of subsequent system failure after detection and repair. The term considers that the post-repair capacity of member(s) j may be different (generally lower) from that of the (presumed) initial capacity. Uncertainties may be larger, in particular. The term allows for the (reduced) remaining (expected) life in this "weakened" state, and hence less severe loads. (The exposure times $\bar{T}_j$ and $L_{R_{ij}}$ will be treated out of sequence, for clarity.)

$\bar{D}_j$ This term "corrects" the base member case from $p_{DL} \cdot O \cdot D_0$, the annual* failure probability of a fully-code-stressed member (if D_0=1) or generally from the most-likely-to-fail member in the intact structure, used in $p_{f_{OL}}$, to $p_{DL} \cdot O \cdot \bar{D}_j$, the annual probability of failure of the most heavily stressed member in the damaged structure (e.g., with member j missing). $\bar{D}_j$ allows for stress increases to ***overstressed conditions*** under the design load. $\bar{D}_j$ might have a value of 10^1 or higher if member j was a critical member. If, however, the damaged member was not important to the lateral load carrying capacity of the system, $\bar{D}_j$ may be as low as one, i.e., the base member remains the member stressed only to the code limit.

I^Q_{ij} An ***inspection quality factor:*** the probability of detecting the damaged member(s) j given the policy for post-scenario i inspection (if any; the occurrence of of the scenario itself may go undetected, as in the construction or transportation flawed member). $1-I^Q_{ij}$ is the non-detection probability.

$\bar{C}_j$ This factor is equivalent to C_0 except it is for the damaged structure (e.g., member j missing). If stress increases (as reflected by $\bar{D}_j$) are localized to the vicinity of the damaged member, $\bar{C}_j$ may be much less than C_0, perhaps close to one, implying, for example, it is virtually certain that the single most-stressed member will fail first.

*Strictly, it corrects the failure probability over the exposure time $\bar{T}_j$. The exposition is simpler here in annual terms and the numerical differences may often be small.

$\overline{R}_j$ This factor is equivalent to R_0 except it is for the damaged structure. It is very possible that the net *damaged-structure system factor*, $\overline{C}_j \overline{R}_j$, is less (better) than that of the intact, $C_0 R_0$, because the damage localizes the most highly stressed members (the stress increase per se is covered by $\widetilde{D}_j$) and because of the relatively larger available capacities in other members if yet another member should fail. $p_{DL} O\overline{D}_j \overline{C}_j \overline{R}_j$ is the annual probability of failure of the damaged system. (Note the term M_S to correct for the implicit static pushover model basis; this factor might also depend on j in some cases, e.g., when the damage reduces system ductility.)

$\overline{T}_j$ The (expected) time during which the structure is exposed to the environment (e.g., wave and seismic) in its (undetected) damaged state. If the next regular inspection is certain to find the damage, this time is just $T/2$, where T is the interval between regular inspections (a policy matter). (The 1/2 reflects randomness of the occurrence time of the scenario within T.) More generally, $\overline{T}_j$ is given by Eq. A.4, which includes allowance for imperfect inspection increasing this exposure time. If all I^Q_{0j} are zero, $\overline{T}_j = L/2$.

$\widetilde{I}^Q_{ij_{net}}$ *An inspection quality factor*. If all regular inspections were perfect (i.e., all $I^Q_{0j}=1$), then this factor would be just I^Q_{ij}. As shown in Eq. A.5, this net inspection factor is, strictly, the probability of detection of the damage j prior to failing in the damaged state. Provided $p_{f_{damaged,ij}}$ is small relative to one, we can approximate $\widetilde{I}^Q_{ij_{net}}$ by simply the probability that the damage is *ever* detected (Eq. A.5.) If this approximation does not hold Eq. A.5 must be modified by factors similar to those in Eq. A.4 to allow for various possibilities as to how long the structure is exposed in the damaged state before detection.

$\widetilde{C}_j$ Similar to C_0 but for a repaired system. It allows for the fact that the repaired member(s) may have different (higher, typically, perhaps) failure probabilities than they did when intact (or presumed to be intact in the case of a poorly constructed member). If the repaired members are not highly stressed under waves, $\widetilde{C}_j$ may be virtually identical in value to C_0. On the other hand, $\widetilde{C}_j$ will increase (typically) if the repaired members were critical to the system.

$\tilde{R}j$ Similar to R_0 but for a repaired system (see $\tilde{C}j$). It may not be significantly different from R_0 in value, but it will correct, for example, for possibly revised relative likelihoods of member-failure sequences. (Again if the repair affects system ductility M_S may also change for the repaired system.)

$L_{R_{ij}}$ The exposure time in the repaired state, i.e., the economic life remaining after repair. It equals L minus the (conditional) expected time to occurrence of scenario i (given that it occurs), which can range from zero for construction errors, through $L/2$ for random accidents, to perhaps almost L for fatigue/fracture scenarios and minus $\overline{T}_j$, the expected time to detection and repair. (Strictly D_0 in Eq. A.3 should correct not for the lifetime but for $L_{R_{ij}}$ and hence will depend to some (often mild) degree on i and j.)

I^{Q}_{0j} Inspection quality factor in regular (i.e., not post-accident, or here, "post-scenario") inspection. See I^{Q}_{ij}. In principle, I^{Q}_{0j} should be the detection probability *given* no post-scenario detection, and hence depends on i.

$I^{Q}_{0j,2}$ Inspection quality factor in the second regular inspection; it is the probability of detecting member j damage *given* it was not detected the first time. It may be nearly zero. (Again, it should in principle also be conditioned on no post-scenario detection.)

APPENDIX B
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