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COMPUTER ANALYSIS OF THE CONSOLIDATION OF OCEAN BOTTOM SEDIMENT

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ABSTRACT

Consolidation equation and energy equations were derived in [13]. The first effort is to provide reliable, efficient codes for the solution of these two equations. Inspection of these equations shows that the consolidation equation can be solved independent of the energy equation. The solution of the energy equation requires the solution of the consolidation equation. This system is solved using a divided difference scheme for the spatial derivatives and a backward difference scheme for the time derivative. The resulting set of tridiagonal equations is easily solved.

The system adequately models laboratory tests. The energy equation can be ignored and only the consolidation equation need be solved with boundary conditions related to the applied pressure.

The model gives no consolidation for the boundary condition of field data. This has led to derivation of a more complete model and development of a different scheme for their solution.

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1. INTRODUCTION

1.1 Objective

We would like to solve the equations modelling the phenomena of consolidation of a soil such as the sediment at the bottom of the ocean. Further, we would like to allow additional deposition with time.

The equations are coupled sets of nonlinear partial differential equations in one spatial coordinate (depth) and time. We do not expect the availability of analytical solutions except in limited cases.

1.2 Statement to the Problem

The following consolidation and energy equations for marine sediments are taken from [13]:

$$(1.2.1) \qquad Q(n)\,\frac{\partial^2 n}{\partial z^2} + R(n)\left(\frac{\partial n}{\partial z}\right)^2 + S(n)\,\frac{\partial n}{\partial z} = \frac{\partial n}{\partial t}$$

Energy equation:

$$(1.2.2) \qquad \frac{\partial \theta}{\partial t} = A_1\,(n,\theta)\,\frac{\partial^2 \theta}{\partial z^2} + A_2\,(n,\theta)$$

where $Q(n)$, $R(n)$, $S(n)$ are functions of the porosity, variable with depth and time; $A_1(n,\theta)$ and $A_2(n,\theta)$ are functions of porosity, n, and temperature, θ.

We have two cases for solving equations (1.2.1) and (1.2.2):

the laboratory case and the field case. These cases have the following boundary conditions:

For the laboratory case:

(1.2.3) $n = f(\text{pressure})$ at $z = 0$ and $z = L$.

For the field case:

(1.2.4) $n(z = 0) = 0.85$ for all t

(1.2.5) $\frac{\partial n(z = L)}{\partial z} = 0$ for all t.

Note: The limits of the variable z are zero and L. The lower limit is the bottom of the sample in the laboratory case and the basement in the field case. The upper limit is the thickness of the laboratory sample or the height of the sediment, the interface between the sea and the formation.

2. METHODS OF SOLUTION

2.1 INTRODUCTION

The methods of solving the equations introduced in the previous chapter include analytical techniques, numerical methods, and limiting case analysis.

2.2 ANALYTICAL TECHNIQUES

The detailed form of the equations presented in (13) immediately shows that there is a small probability of solving the equations in closed form. This leaves the value of analytical techniques to be a general understanding of the expected behavior of the solution. This will come from understanding the solution of similar linear partial differential equations which are more easily solved.

2.3 NUMERICAL METHODS

It is generally expected that the solution of problems of this difficulty will be solved or "simulated" through the use of numerical analysis. The usual procedure is to replace the spatral derivatives with "central" finite difference approximations and time derivatives with "forward" or "backward" differences to yield explicit and implicit formulae, respectively. Chapter 3 of this report is the details of a finite difference approach to the solution of this model.

2.4 LIMITING CASE ANALYSIS

A useful part of theoretical analysis of similar models is to consider the solution as time becomes arbitrarily large. In this limiting case, all derivatives with respect to time become zero as the model reaches "steady state". This simplifies the model because the partial differential equations become ordinary differential equations.

The use of the one dimensional maximum principle (see Protter & Wernberger (9)) can be used to show that the model with the field boundary conditions admits only the "unchanged" limiting case solution. This leads to the conclusion that the model is inadequate for the field case. This does not affect its adequacy for the laboratory case. The following analysis applies to the field case:

A function u (x) that is continuous on the closed interval (a,b) takes on its maximum at a point on this interval. If u (x) has a continuous second derivative, and if u has a relative maximum at some point c between a and b, then we know from elementary calculus that

(2.4.1) $u'(c) = 0$ and $u'(c) \leq 0$

Suppose that in an open interval (a,b), u is known to satisfy a differential inequality of the form

(2.4.2) $L(u) = u'' + g(x)\, u' > 0.$

where g(x) is any bounded function. Then it is clear that relations (2.4.1.) cannot be satisfied at any point c in (a,b). Consequently, wherever (2.4.2.) holds, the maximum of u in the interval cannot be attained anywhere except at the endpoints a or b. We have here the simplest case of a maximum principle.

An essential feature of the above argument is the requirement that the inequality (2.4.2.) be strict; that is, we assume that $u'' + g(x)\, u'$ is never zero. In the study of differential equations and in many applications, such a requirement is overly restrictive, and it is important that we remove it if possible. We note, however, that for the nonstrict inequality

$$u'' + g(x)\, u' \geq 0,$$

the solution u = constant is admitted. for such a constant solution

the maximum is attained at every point. Protter [8] has proved that this exception is the only one possible.

We can repeatedly apply the maximum principle to prove that the Degenerated Consolidation Equation has a constant solution only.

2.5 Uniqueness of the Solutions of the Boundary Value Problem

Keller [6] gives complete discussions of the applicability of several numerical methods for the solution of two point boundary value problems. The equation [4.3.1.] and [4.3.2] and their associated boundary conditions give us confidence that the procedures outlined as in (9) give unique solutions of the boundary value problem.

2.6 Parameter Estimation in Quasilinear Parabolic Equations - Oil Reservoir Applications

The method utilizes Newton-Raphson method, finite differences and least squares fitting criteria to estimate the parameters appearing in systems of Quasilinear Parabolic Partial Differential Equations is described in [12]. Numberical results [12] indicate that the Newton-Raphson method yields accurate estimates for the parameters within a reasonable number of iterations. It is also found that the Newton-Raphson method is an efficient method to estimate parameters in partial differential equations.

The methods developed by StuckenBruck [12] and Childs, et.al. [3] can be used with those under development on this project to determine the "best fit" soil parameters to make the model fit the laboratory data.

3. A STATIC CONSOLIDATION EQUATION AND ENERGY EQUATION

3.1 A finite Difference Representation of the Model

The consolidation equation (1.2.1) defines the relationship of the porosity function with respect to depth and time.

This equation can be solved by a Picard iteration. This is set up by using the explicit derivatives as unknowns and the coefficients on the left hand side as known functions of the current estimate of the solution. The solution for each time step converges in two or three interations in most cases.

3.2 Some Sample Output of the Solution of the Consolidation Equation and the Energy Equation for the Laboratory Case

As in Figure 1, we take N = 100 and divide the sample height into 99 equal parts, each part is Δz. Porosity (i) denotes the porosity at point i. The input is shown in Table 1. Then the output statistics are shown in Table 2.

By using Table 2 we get the Figure 3, which shows the process of consolidation. At time T = 0 minute, we have initial porosity 0.337 at point 1, 16, 33, 50, 66, 83 and 100; at T = 20 minutes, we have a slight consolidation; at T = 200 minutes, we have better consolidation; and at T = 500 minutes we have a reasonable approximately perfect consolidation.

By using Table 4 we get the Figure 4. It shows clearly from Figure 4 that time starts from T = 0 minute, the porosity at each point stays at initial porosity, i.e., 0.5584. As time goes on, the consolidation process is going on, and at T = 330 minutes we get a reasonable perfect consolidation.

For illite under 1520 psi, temperature at 20 degrees centigrade, we have the experiment results for the time versus sample height during the process of consolidation as shown in Table 5.

A finite difference scheme is used to solve the consolidation equation. Figure 1 shows the nadal points at which the porosities are chosen as discrete unknowns and are to be evaluated. In the Lagrangean formulation the finite differences vary with depth and time and are dependent on the local instantaneous value of the porosity.

For any point i, Equation (1.2.1) may be written as follows:

$$Q_i(n)\left[\frac{\partial^2 n}{\partial z^2}\right]_i + R_i(n)\left[\frac{\partial n}{\partial z}\right]_i^2 + S_i(n)\left[\frac{\partial n}{\partial z}\right]_i = \left[\frac{\partial n}{\partial t}\right]_i$$

where all the coefficients and derivatives refer to point i under consideration.

Note that for the field case: the upper boundary corresponds to the bottom of the ocean. If the lower boundary is fixed, it corresponds to the basement.

Let us first express the derivatives of porosity function in terms of the dependent variable at the nodal points. For convenience we denot

$$a = (\Delta z)_i \text{ and } b = (\Delta z)_{i+1}$$

in Figure 2.

Table 1

Input data for illite run at $20^{o}C$

Sample	Sample Height HCM	Consolidation		Permeability	
Illite	1.962139 cm	Constants		Constants	
		A (KG/cm^2)	B	C (cm/sec)	D
		.66232502	- 6.0804348	.546E-06	8.4
		Initial Porosity .337	Initial temperature 20 degrees centigrade	Consolidation pressure 106.4 kg/cm^2 (1520 psi)	Increment time step 20 sec

Table 2

Output at 1/3 points for illite run at 20°C

Time (Min)	Porosity (0)	Porosity (33)	Porosity (67)	Porosity (100)
t = 0	.3370	.3370	.3370	.3370
t = 20	.2970	.3138	.3294	.3345
t = 200	.2970	.2990	.3008	.3015
t = 500	.2970	.2971	.2972	.2473

4. THEOREMS AND SOLUTIONS RELATED TO THE CONSOLIDATION EQUATION AND THE ENERGY EQUATION

4.1 Theorems Related to the Solutions to Degenerated Consolidation Equation

There are two theorems [9] related to Degenerated Consolidation Equation can be given as follows:

Theorem 4.1.1 (One-dimensional maximum principle). Suppose $u = u(x)$ satisfies the differential inequality.

$L(u) \equiv u'' + g(x)u' \geq 0$ for $a < x < b$, with $g(x)$ a bounded function. If $u(x) \leq M$ in (a, b) and if the maximum M of u is attained at an interior point c of (a, b), then $u \equiv M$.

Theorem 4.1.2 Suppose u is a nonconstant function which satisfies the inequality $u'' + g(x)\, u' \geq 0$ in (a, b) and has one-sided derivatives at a and b, and suppose g is bounded on every closed sub-interval of (a, b). If the maximum of u occurs at $x = a$ and g is bounded below at $x = a$, then $u'(a) < 0$. If the maximum occurs at $x = b$ and g is bounded above at $x = b$, then $u'(b) > 0$.

4.2 Theorems Related to the Existence and Uniqueness Solution of the Approximating the Partial Differential Equations - Consolidation Equation and Energy Equation

The differential equation of order n

$$\frac{d^n y}{dx^n} = f\left(x, y, \frac{dy}{dx}, \ldots, \frac{d^{n-1}y}{dx^{n-1}}\right)$$

is equivalent to the system of first-order equations

$$\frac{dy}{dx} = y_1$$

$$\frac{dy_1}{dx} = y_2$$

$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$

$$\frac{dy_{n-2}}{dx} = y_{n-1}$$

$$\frac{dy_{n-1}}{dx} = f(x, y, y_1, \ldots, y_{n-1})$$

Equivalence means that any solution of the system defines a solution of the equation, and conversely. In fact any system of differential equation of order greater than 1 may be reduced to a first-order system by first solving explicitly for the highest-order derivative. This process may lead to certain difficulties, e.g., extracting roots [10].

If, in the equations of a first-order system, the independent variable t appears explicitly on the right side, then we write

$$\dot{x}_i = f_i(x_i, \ldots, x_n, t) \quad i = 1, \ldots, n$$

which is called a nonautonomous system. Without the explicit appearance of t we have

$$\dot{x}_i = f_i\,(x_i, \ldots, x_n) \quad i = 1, \ldots, n$$

which is known as an autonomous system. Note that a nonautonomous

system can be reduced to a special autonomous system by replacing t on the right by the dependent variable x_{n+1} and adjoining to the resulting system the equation

$$\frac{dx_{n+1}}{dt} = 1.$$

Theorem 4.2.1 Let the function f_i be continuous in a domain D (a rectangle) defined by

$$|t - t_o| < a \qquad |x_i - x_i^o| < a_1 \qquad i = 1, \ldots, n$$

and let the following Lipschitz condition be satisfied for any two points $\bar{x}$ and $\bar{\bar{x}}$ (with the same value of t) in D:

$$|f(t, \bar{x}) - f(t, \bar{\bar{x}})| < \sum_{j=1}^{n} A_j |\bar{x}_j - \bar{\bar{x}}_j|$$

Then in a suitable interval $|t - t_o| < \bar{a} < a$, the system has a unique solution $x_i(t)$ which satisfies $x_i(t_o) = x_i^o$, $i = 1, \ldots, n$ [10].

4.3 The Existence and Uniqueness Solution of the Approximating the Partial Differential Equations - Consolidation Equation and Energy Equation

The complete proof of existence and uniqueness of the coupled partial differential equations (i.e., the consolidation equation and the energy equation) is quite difficult at best.

We will refer to [9] for related results. For the consolidation

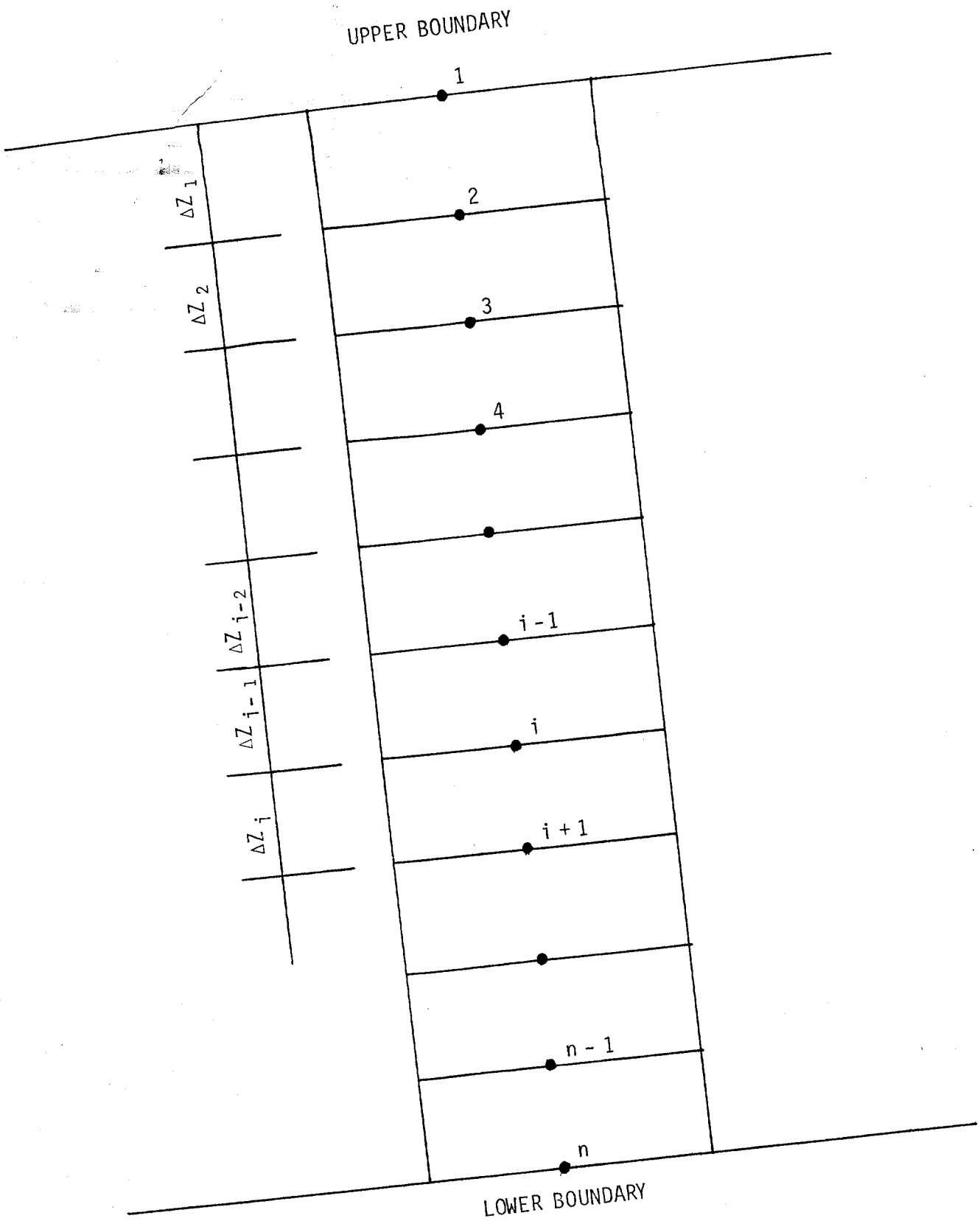

Fig. 1. Finite difference scheme

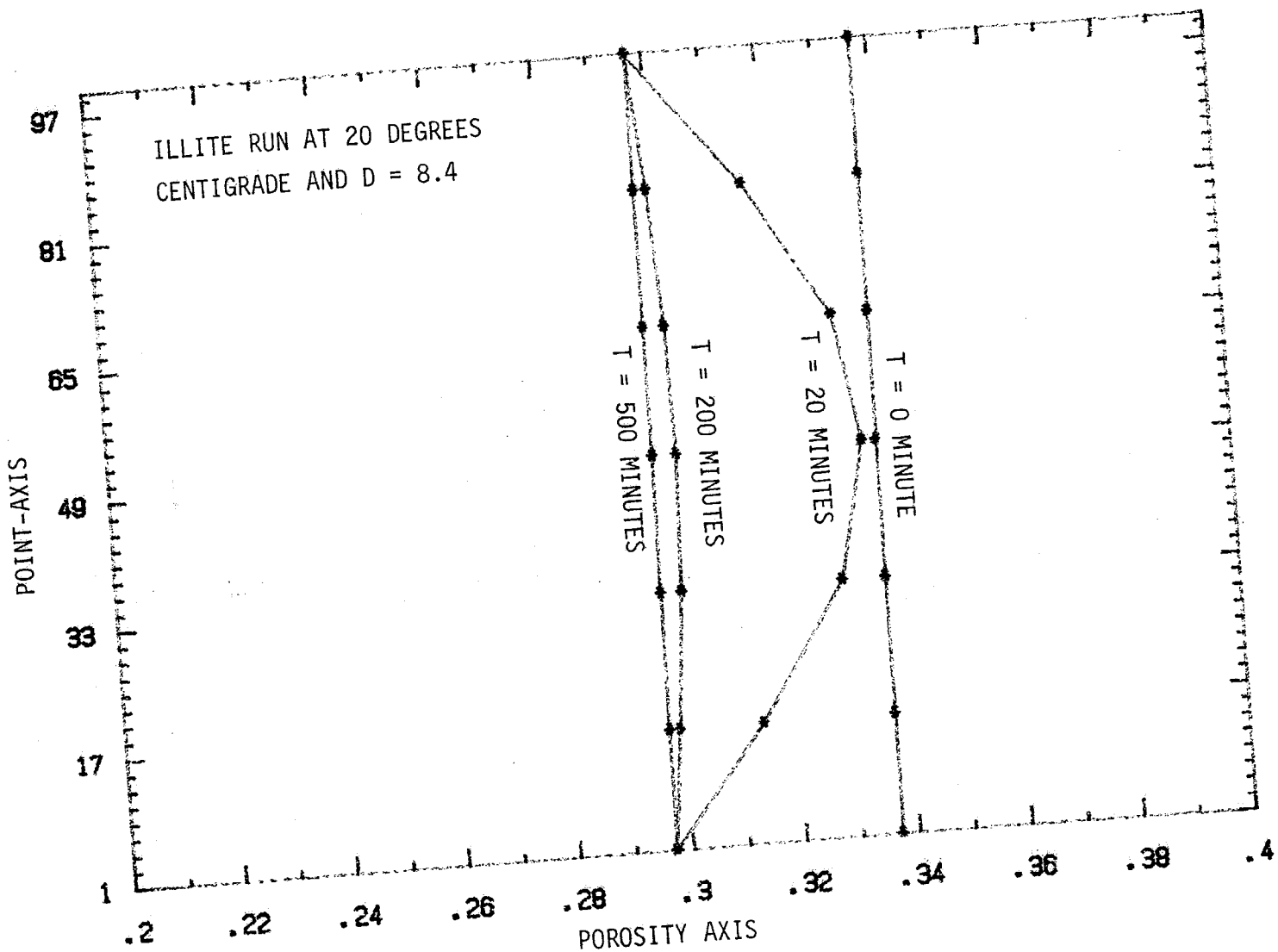

Fig. 3. Porosity versus time at different points

Table 3

Input data for bentonite run at 90°C

Sample	Sample Height HCM	Consolidation		Permeability	
bentonite	2.149099 cm	Constants		Constants	
		A (kg/cm^2)	B	C (cm/sec)	D
		0.4922999	- 5.5637999	.522E-07	8.89
		Initial porosity P INIT = .5584	Initial temperature TMI = 90 degrees centigrade	Consolidation pressure PO = 25.2 kg/cm^2 (360 psi)	Increment time step DT = 10 sec

Table 4

Output at 1/3 points for bentonite run at 90°C

Time (min)	Porosity (0.0)	Porosity (1/3)	Porosity (2/3)	Porosity (1.0)
t = 0	.5584	.5584	.5584	.5584
t = 20	.4930	.5300	.5539	.5579
t = 60	.4930	.5144	.5361	.5450
t = 100	.4930	.5087	.5247	.5315
t = 160	.4930	.5037	.5141	.5184
t = 200	.4930	.5014	.5094	.5127
t = 250	.4930	.4993	.5052	.5076
t = 330	.4930	.4971	.5007	.5022

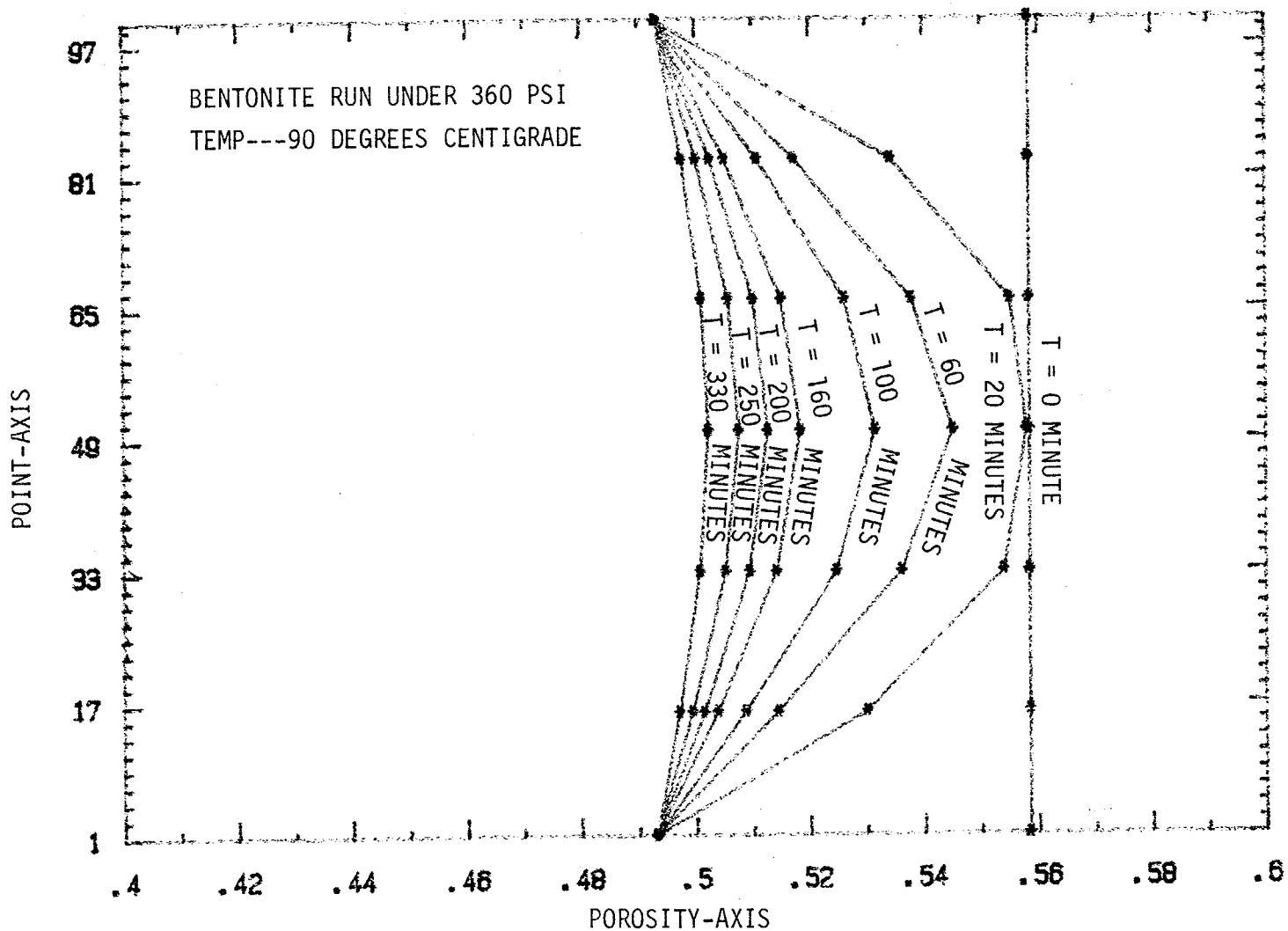

Fig. 4. Porosity versus time at different points

Table 5

Experiment results for illite under 1520 psi

Time (minutes)	Sample height (inches)
0.1	0.7725
0.2	0.7710
0.4	0.7690
0.7	0.7680
1	0.7660
2	0.7640
4	0.7600
7	0.7560
10	0.7530
20	0.7470
40	0.7390
70	0.7360
100	0.7335
900	0.7286
920	0.7286
930	0.7286
940	0.7286
950	0.7286
960	0.7286
970	0.7286
980	0.7286
1000	0.7286

Table 6

Output for illite run under 1520 psi and DT = 1

Time (minutes)	Sample height (inches)
0.1	0.7725
0.167	0.7683
0.5	0.7651
1	0.7620
2	0.7576
3	0.7543
4	0.7515
5	0.7492
10	0.7417
20	0.7349
30	0.7319
40	0.7304
50	0.7296
60	0.7292
70	0.7289
80	0.7288
90	0.7287
100	0.7287
110	0.7286
120	0.7286
130	0.7286

Table 7

Output for illite run under 1520 psi and DT = 10

Time (minutes)	Sample height (inches)
0.1	0.7725
0.167	0.7688
0.5	0.7656
1	0.7624
2	0.7579
3	0.7545
4	0.7518
5	0.7495
10	0.7418
20	0.7349
30	0.7319
40	0.7304
50	0.7295
60	0.7291
70	0.7288
80	0.7287
90	0.7286
100	0.7286
110	0.7285
120	0.7285
130	0.7285
140	0.7285

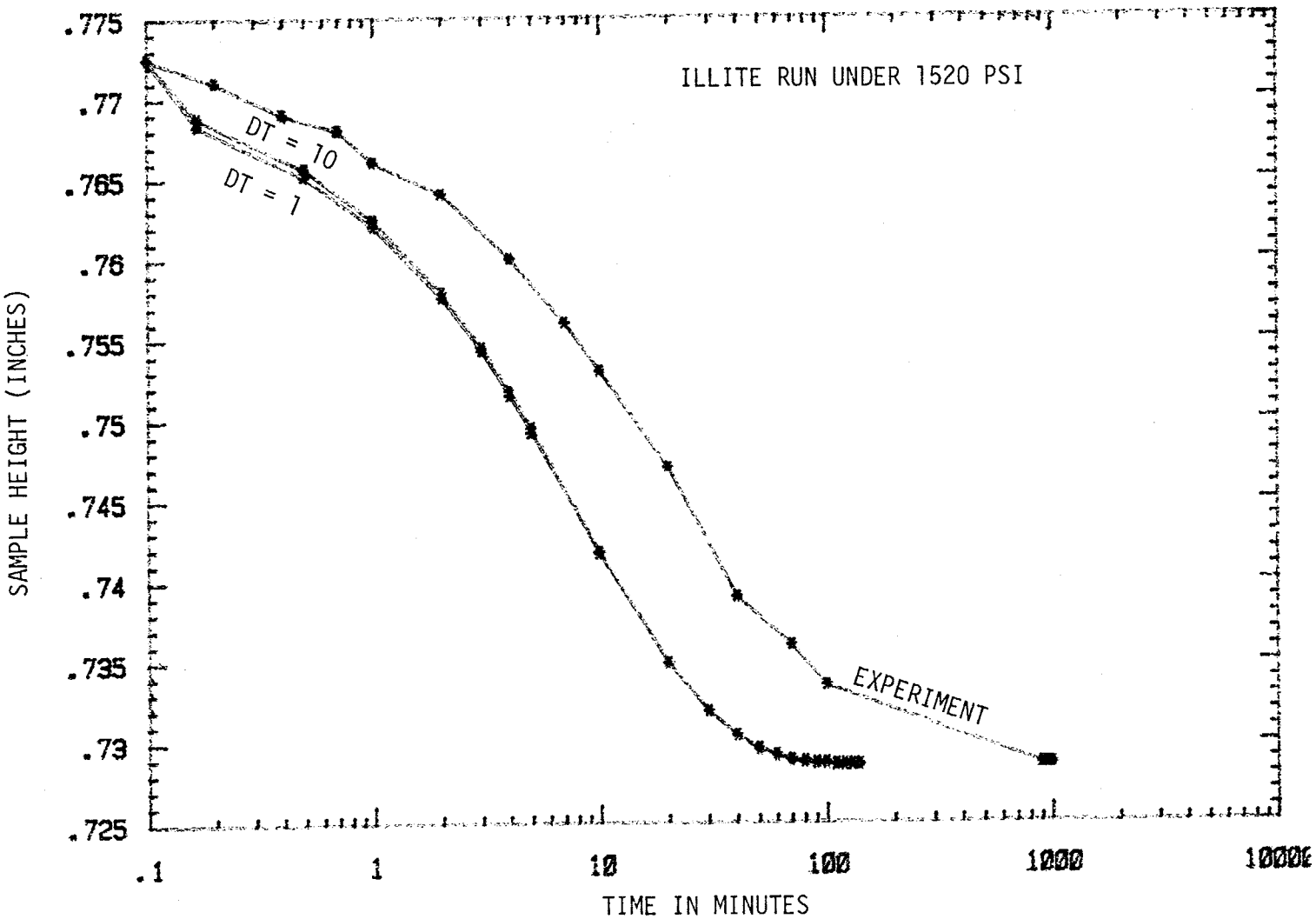

Fig. 5. Time versus sample height

Table 8

Output for illite run under 1520 psi and DT = 50

Time (minutes)	Sample Height (inches)
0.1	0.7725
5	0.7503
10	0.7425
15	0.7381
20	0.7353
25	0.7334
30	0.7321
35	0.7312
40	0.7305
45	0.7300
50	0.7296
55	0.7293
60	0.7291
65	0.7290
70	0.7288
80	0.7287
90	0.7286
95	0.7286
100	0.7285
105	0.7285
110	0.7285
115	0.7285
120	0.7285

Table 9

Output for illite run under 1520 psi and DT = 100

Time (minutes)	Sample height (inches)
0.1	0.7725
5	0.7511
10	0.7432
15	0.7386
20	0.7358
25	0.7338
30	0.7324
35	0.7314
40	0.7307
45	0.7302
50	0.7298
55	0.7295
60	0.7293
65	0.7291
70	0.7289
80	0.7388
90	0.7388
95	0.7286
100	0.7286
105	0.7286
110	0.7286
115	0.7286
120	0.7286

Table 10

Output for illite run under 1520 psi and DT = 200

Time (minutes)	Sample height (inches)
0.1	0.7725
3.333	0.7566
10	0.7443
20	0.7366
30	0.7330
40	0.7311
50	0.7300
60	0.7294
70	0.7291
80	0.7289
90	0.7287
100	0.7287
110	0.7286
120	0.7286

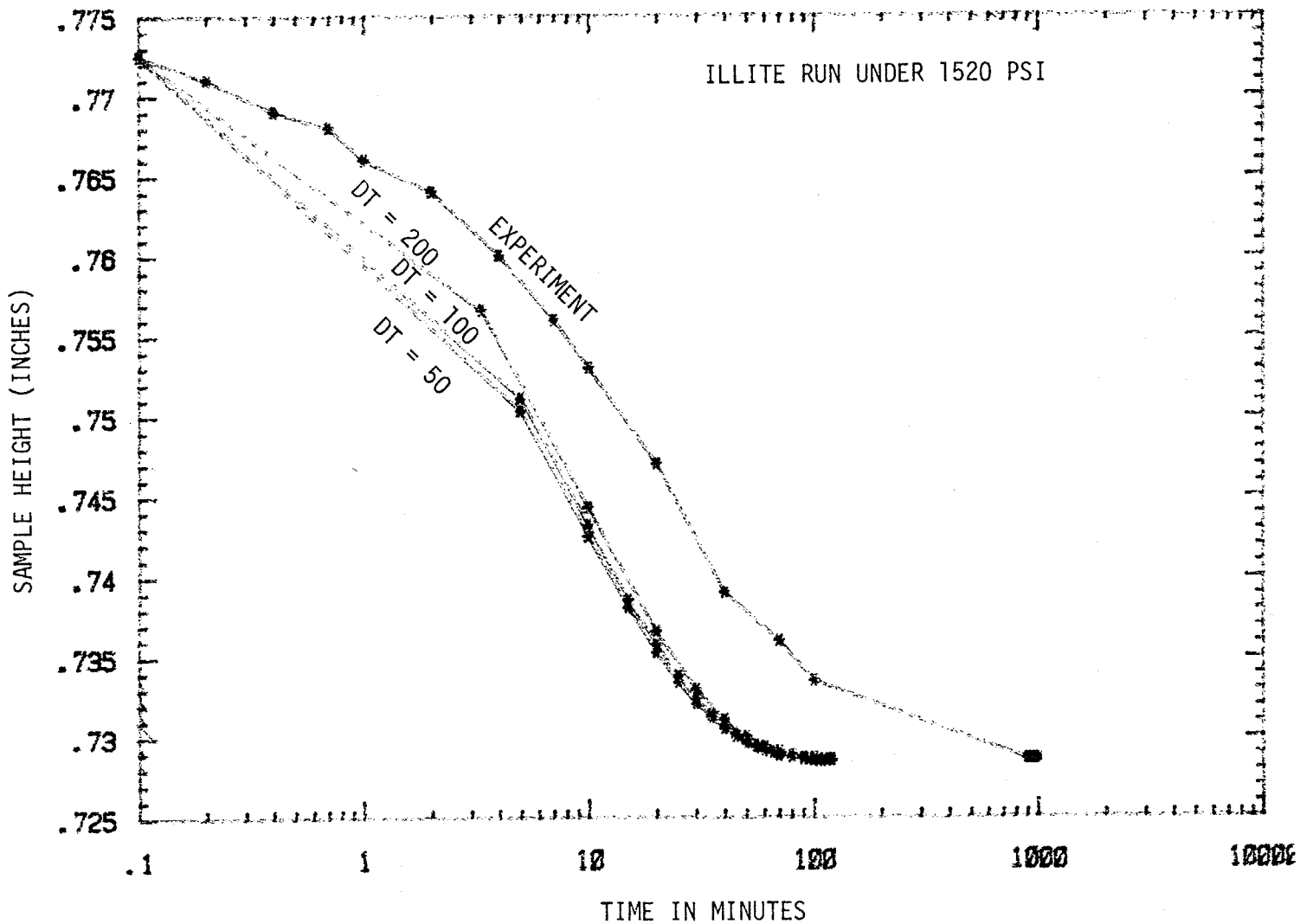

Fig. 6. Time versus sample height

equation, we first replace the time derivative by a backward difference formula:

$$\frac{\partial n}{\partial t} = \frac{n(t) - n(t - \Delta t)}{\Delta t}$$

and approximate the square of the spatial derivative by $(\partial n/\partial z)^2 = (\partial n(t - \Delta t)/\partial z)(\partial n(t)/\partial z)$. This gives a linear ordinary differential equation in terms of $n(t)$ if we use $(t - \Delta t)$ in the coefficients of the consolidation equation we then have:

$$(4.3.1) \qquad Q(n(t-\Delta t)) \frac{\partial^2 n(t)}{\partial z^2} + \left\{ R(n(t-\Delta t)) \frac{\partial n(t-\Delta t)}{\partial z} \right\} \frac{\partial n(t)}{\partial z} +$$

$$S(n(t-\Delta t)) \frac{\partial n(t)}{\partial z} = \frac{n(t) - n(t-\Delta t)}{\Delta t}$$

We can do likewise with the energy equation giving:

$$(4.3.2) \qquad A_1 (n(t-\Delta t), \theta(t-\Delta t)) \frac{\partial^2 \theta(t)}{\partial z^2} + A_2 (n(t-\Delta t), \theta(t-\Delta t))$$

$$= \frac{1}{\Delta t} (\theta(t) - \theta(t-\Delta t))$$

Further, we can write (4.3.1) and (4.3.2) as four first order equations:

$$\frac{dn}{dz} = m$$

$$\frac{dm}{dz} = \left(\frac{1}{Q(n(t-\Delta t))} (-R(n(t-\Delta t)) m(t-\Delta t) m(t) -\right.$$

$$\left. S(n(t-\Delta t)) m(t) + (n(t) - n(t-\Delta t))/\Delta t\right)$$

$$\frac{d\theta}{dz} = \phi$$

$$\frac{d\phi}{dz} = \frac{1}{A_1(n(t-\Delta t),\ \theta(t-\Delta t))} \Big(- A_2(n(t-\Delta t),\ \theta\ (t-\Delta t)) + \frac{1}{\Delta t}\ (\theta(t) - \theta(t-\Delta t))\Big)$$

It is obvious that this set of equations meets the condition necessary for theorem 4.2.1, hence, by theorem 4.2.1, has a unique solution. This unique solution implies existence and uniqueness solution for (4.3.1) and (4.3.2).

4.4 Analytic Solutions

The consolidation and energy equations have analytic solutions reasonably available only after discarding many terms. Such solutions can be obtained through the usual separation of variables techniques if the following assumptions are made:

$R(n) = 0$

$Q(n) = \text{constant}$

$S(n) = \text{constant}$

$A_1(n, \theta) = \text{constant}$

$A_2(n, \theta) = 0$

This is clearly too restrictive.

4.5 A Limiting Case of the Consolidation Equation

A limiting case of the consolidation equation is obtained letting $t = \infty$. All derivatives with respect to t then approach zero.

THE CONSOLIDATION EQUATION AS $t \to \infty$:

The consolidation equation in (1.2.1) is

$$(1.2.1) \qquad -\frac{A(B-1)C}{G_w \gamma_o} n^{B+D-2} \frac{\partial^2 n}{\partial z^2} + CD \frac{G_s}{G_w} n^{D-2} (1-n-D^{-1}) \frac{\partial n}{\partial z} = \frac{\partial n}{\partial t}$$

So as $t \to \infty$, the consolidation equation degenerates to:

$$(4.5.1) \qquad -\frac{A(B-1)C}{G_w \gamma_o} n^{B+D-2} \frac{\partial^2 n}{\partial z^2} + CD \frac{G_s}{G_w} n^{D-2} (1-n-D^{-1}) \frac{\partial n}{\partial z} = 0$$

This implies

$$(4.5.2) \qquad \frac{d^2 n}{dz^2} + \gamma\{\beta - n\} \; n^{\alpha} \frac{dn}{dz} = 0$$

The above equation is subject to the boundary conditions

$$(4.5.3) \qquad n(o) = n_o \text{ and } n'(L) = 0,$$

which are equivalent to (1.2.4) and (1.2.5).

Theorem 4.5.1 If $n(z)$ is a solution to

$$\frac{d^2 n}{dz^2} + g(z) \frac{dn}{dz} = 0, \quad 0 \le z \le L,$$

where $g(z) = \gamma \{\beta - n\} n^{\alpha}$; $\quad n(o) = n_o$

and $n'(L) = 0$ and $n(z)$ is a bounded function on

$[0, L]$. Then $n(z)$ is a constant.

Before prove theorem 4.5.1, we have to list two theorems - theorem 4.5.2 and theorem 4.5.3. By [9], we have these two theorems as follows:

<u>Theorem 4.5.2</u> If $n(z)$ has a maximum on $(0, L)$, then $n(z) =$ constant.

<u>Theorem 4.5.3</u> If the maximum of $n(z)$ occurs at L, then $n'(L) > 0$.

Proof of Theorem 4.5.1:

Suppose $n(z)$ is a nonconstant solution, we are going to prove that we will get a contradiction, thus $n(z)$ must be constant.

If $n(z)$ is not a constant, then by theorem 4.5.2, $n(z)$ has no maximum on $(0, L)$.

By theorem 4.5.3 and the given condition $n'(L) = 0$ imply that the maximum of $n(z)$ can not occur at $z = L$.

Conclude that if $n(z)$ is not constant, then the maximum of $n(z)$ occurs at $z=0$.

So $n(z) \leq n_0$

implies $0 \leq n_0 - n(z)$.

On the other hand,

let $V(z) = n_0 - n(z) \geq 0$

Then $V'(z) = - n'(z)$

$$V''(z) = - n''(z)$$
$$= \gamma\{\beta-n\}\, n^{\alpha}\, n'$$
$$= \gamma\{\beta+V-n_0\}(n_0-V)^{\alpha}\,(-V').$$

Hence $V'' + h(z)\, V' = 0$ with

$$V = 0 \text{ and } V'(L) = 0,$$

where $h(z) = \gamma\{\beta+V-n_0\}(n_0-V)^{\alpha}$.

Again, we cannot have the maximum of V at $z = L$ since $V'(L) = 0 \not> 0$; and if the maximum of $V(z)$ occurs on $(0, L)$ then $V = \text{const} = 0$; so

unless V is constant, maximum V can only occur at $z = 0$.

implies $V(z) \leq 0$ since $V(0) = 0$

implies $n_0 - n(z) \leq 0$

implies $n_0 \leq n(z)$.

But we have $n(z) \leq n_0$

so $n(z) \equiv n_0$, which is a constant.

Hence we obtain a contradiction.

Therefore $n(z)$ is a constant. $\square$

Theorem 4.5.1 tells us that there is no reason to make further research for the field case. From experiment and program output we know that the temperature keeps constant in laboratory cases.

Thus, we can see the consolidation equation (1.2.1) subject to the field type boundary conditions cannot give a "consolidated" solution. This quasistatic model is thus inadequate.

5. A DYNAMIC CONSOLIDATION MODEL

5.1 Introduction

Dr. Thompson has derived a set of equations including dynamic terms. He has assumed this is not a shock wave. That is, he has assumed that at the instant a load is applied at the surface, it is uniformly applied over the entire depth of the formation.

The equations are:

Equation of motion of water -

$$(5.1.1) \qquad -\frac{\partial un}{\partial z} - \tau - nG_w\gamma_o = \frac{dv_w nG_w \frac{\gamma_o}{g}}{dt}$$

Equation of motion of solids -

$$(5.1.2) \qquad -\frac{\partial An^B}{\partial z} - h(1-n)\frac{\partial u}{\partial z} + \tau - (1-n)\,G_s\gamma_o = \frac{dv_s(1-n)\,G_s\,\frac{\gamma_0}{g}}{dt}$$

Rate of loading -

$$(5.1.3) \qquad \frac{\partial[An^B + u(n+h(1-n))]}{\partial t} = \text{rate of loading}$$

Conservation of mass of solids

$$(5.1.4) \qquad \frac{\partial((1-n)G_s)}{\partial t} + \frac{\partial(V_s(1-n)G_s)}{\partial z} = 0$$

Conservation of mass of water

$$(5.1.5) \qquad \frac{\partial(nG_w)}{\partial t} + \frac{\partial(V_w nG_w)}{\partial z} = 0$$

Darcy equation -

$$(5.1.6) \qquad Cn^D\left(1 + \frac{1}{G_w\gamma_o}\frac{\partial u}{\partial z}\right) G_w + V_w nG_w - V_s(1-n)G_w = 0$$

where n is porosity,

u is pore pressure,

V_s is the velocity of solids,

V_w is the velocity of water,

and the rest are the same as before.

The equations of motion are combined by eliminating τ to give:

$$(5.1.7) \qquad \frac{\gamma_0}{g}\frac{\partial}{\partial t}(G_s\ V_s\ (1-n)) +$$

$$\frac{\gamma_0}{g}\frac{\partial}{\partial t}(nG_w V_w) + \frac{\partial[An^B + u(n+h(1-n))]}{\partial z}$$

$$+ (1-n)\ G_s\gamma_o + nG_w\gamma_o = 0$$

The approach to solving these equations is based on the assumption that the functions n, u, V_s and V_w can be represented with low order polynomials at specified values of t. We can then use a finite difference approximation for derivatives with respect to t.

At time $t = t$,

(5.1.8) $n = p_1(z)$

where $p_1(z) = a_0 + a_1 z + \ldots + a_k z^k$

and a's are unknown coefficients.

(5.1.9) $u = p_2(z)$

where $p_2(z) = b_0 + b_1 z + \ldots + b_k z^k$

and b's are unknown coefficients,

(5.1.10) $V_s = p_3(z)$

where $p_3(z) = c_0 + C_1 z + \ldots + c_\ell z^\ell$

and c's are unknown coefficients.

(5.1.11) $V_w = p_4(z)$

where $p_4(z) = d_0 + d_1 z + \ldots + d_\ell z^\ell$

and d's are unknown coefficients.

At time $t = t - \Delta t$,

$$n = P_1(z)$$

$$u = P_2(z)$$

$$V_s = P_3(z)$$

$$V_w = P_4(z)$$

with known coefficients for each $P_i(z)$ for $i = 1, 2, 3, 4$.

First, we substitute these polynomials into the boundary conditions to determine the coefficients in $P_i(z)$. Given L = sediment depth the first set of equations are:

$$n(o, t) = n_o$$

where n_o is the initial porosity i.e.,

$$(5.1.12) \quad n(o, t) = \sum_{i=o}^{3} a_i z^i \Big|_{z=L} = n_o$$

implies

$$(5.1.13) \quad a_0 + La_1 + L^2 a_2 + L^3 a_3 = n_o$$

$$n(z, t) = \sum_{i=o}^{3} a_i z^i \text{ and since}$$

given $\dfrac{\partial n\ (o, t)}{\partial z} = o$ implies

$$\frac{\partial n\ (o, t)}{\partial z} = 1a_1 + 2a_2 z + 3a_3 z^2 \Big|_{z=o}$$

implies

$$(5.1.14) \quad 1a_1 + 0a_2 + 0a_3 = 0$$

$$u(L, t) = \text{depth of ocean} * G_w \gamma_o$$

Since $$u(\lambda) = \sum_{i=0}^{3} b_i \lambda^i,$$

$$u(L) = b_o + b_1 L + b_2 L^2 + b_3 L^3$$

$$= \text{depth of ocean} * G_w \gamma_o$$

i.e.,

(5.1.15) $$1b_o + Lb_1 + L^2 b_2 + L^3 b_3$$

$$= \text{depth of ocean} * G_w \gamma_o.$$

$$V_s(o, t) = o \text{ and}$$

$$V_s(\lambda) = \sum_{i=o}^{2} c_i \lambda^i$$

$$= c_o + c_1 \lambda + c_2 \lambda^2 + c_3 \lambda^3$$

implies

(5.1.16) $$c_o + c_1 o + c_2 o^2 = 0.$$

Given $V_w(o, t) = o$ and

since $V_w(\lambda) = \sum_{i=o}^{2} d_i \lambda^i$, it follows that

(5.1.17) $$d_0 + d_1 o + d_2 o^2 = 0$$

We use differences like

$$\frac{p_i(z) - P_i(z)}{\Delta t}$$

to approximate derivatives with respect to t. The resulting equations can be explicitly differentiated with respect to z. If we use polynomials through 3rd order terms in Equations (5.1.8) and (5.1.9) and through 2nd order terms in (5.1.10) and (5.1.11), then we have 14 coefficients to determine. The boundary conditions (5.1.13) through (5.1.17) give five equations. Additional equations can be written by writing "collocation" equations at different points in z. These equations come from satisfying the governing Equations (5.1.3) through (5.1.7) at these collocation points. If we use four collocation points, we get 20 equations to go with the 5 boundary conditions.

A code from Childs [3] is being used to solve the system of equations such that the boundary conditions exactly and the collocation equations in a least square sense. This is incomplete at the time of this report, but, the investigators are continuing these efforts on their own time.

6. CONCLUSIONS AND DIRECTIONS FOR FUTURE RESEARCH

A code is working for the solution of the static consolidation equations. This code works reasonable for laboratory boundary conditions. It is being adapted to work in a parameter estimation mode such that laboratory data can be used to determine the coefficients in constitutive equations.

Another code is under development for the solution of the dynamic consolidation model. When these codes are completed, an addendum to this report will be forwarded.

APPENDIX A

Transform nonlinear consolidation equation and energy equation into linear equations by backward difference and second order parabola scheme.

Declaration

Input necessary parameters associated with consolidation equation and energy equation.

Output these parameters

While input for time T is not to be cancelled

1. Solve porosities at time T by calling SUBROUTINE TRIDIAG.
2. Set up iterations.

1. Solve temperatures at time T by calling SUBROUTINE TRIDIAG.
2. Set up iteration (one iteration will be enough since energy equation is linear).

Output the solutions for porosities and temperatures.

Fig. 7. Flowchart of a code for solution of solving consolidation equation and energy equation.

APPENDIX B

```
C                                                                      C
C                                                                      C
C     SOLVE CONSOLIDATION EQUATION                                     C
C                 &                                                    C
C        ENERGY EQUATION                                               C
C                                                                      C
C                                                                      C
C     A-----COEFFICIENT FOR THE EFFECTIVE STRESS EXPRESSION.(KG/CM**2)
C           F   = A* N**B; A,B ARE CONSTANTS
C            S
C
C
C     B-----COEFFICIENT FOR THE EFFECTIVE STRESS EXPRESSION.
C           (REAL OR INTEGER).
C
C     PERMEABILITY  K=C* N**D   (CM/SEC)
C     C-----COEFFICIENT FOR THE PERMEABILITY EXPRESSION.
C           (CM/SEC)
C     D-----COEFFICIENT FOR THE PERMEABILITY EXPRESSION.
C
C
C     AREA OF WATER CAN BE EXPRESSED AS A FUNCTION OF THE
C     POROSITY.
C
C
C     A   = N ** E
C      W
C
C     E-----PARAMETER OF THE AREA OF WATER
C     N-----NUMBER OF FINITE DIFFERENCE(NOTE:IN INPUT DATA)
C     DT----THE INITIAL TIME STEP DT (SEC)
C     GS----SPECIFIC GRAVITY OF THE SOLID
C     GW----SPECIFIC GRAVITY OF THE WATER
C     GO----UNIT WEIGHT OF WATER (KG/CM**3)
C     TOL---ABSOLUTE ERROR BOUND FOR THE ITERATIVE SOLUTION
C     H-----INITIAL SAMPLE HEIGHT (CM)
C     TMAX----THE TIME AT WHICH THE CALCULATION SHOULD BE
C           TERMINATED DURING THE CONSOLIDATION PROCESS.(SEC)
C     OM----PARAMETER,0 FOR EXPLICIT SOLUTION,AND 1 FOR IMPLICIT.
C     PO----THE MAXIMUM MAGNITUDE OF THE APPLIED CONSOLIDATION
C           PRESSURE (KG/CM**2).
C     PRVPOR--IS THE POROSITY ON THE PREVIOUS ITERATION
C     PORPTM--IS THE POROSITY ON THE PREVIOUS TIME STEP
C     TMPPTM--IS THE TEMPERATURE ON THE PREVIOUS TIME STEP
C
C     P INIT---INITIAL POROSITY AT ALL POINTS(THE BOUNDARY IS
C           CALCULATED FROM THE KNOWN PRESSURE).
C     SUM DPR------SUM DELTA-POROSITY    (CHANGE THIS ITERATION)
C     BU-----------IS A CONSTANT DEPENDS ON TEMPERATURE WHERE
C       LOG F       =LOG A   +B  LOG N          UNLOADING
C            S MAX        U    U       MIN
C     C   ---------IS A FUNCTION OF N WHERE G C   IS A SPECIFIC
C      VW                                      W VW
C                 HEAT OF THE SEA WATER ONLY IN A UNIT VOLUME OF
C                 THE MINERAL-SEA WATER MIXTURE.
C                 NOTE: G C   IS A CONSTANT.
C                        W VW
C     C   ---------IS A FUNCTION OF N WHERE G C   IS A SPECIFIC
C      VS                                      S VS
C                 HEAT OF THE MINERAL ONLY IN A UNIT VOLUME OF THE
C                 MINERAL-SEA WATER MIXTURE.
C     GG1,GG2,GG3--ARE THE COEFFICIENTS TO DESCRIBE THE MATERIAL.
C                 WHERE K  =GG1 +GG2 *N+GG3 *N**2
```

```
C                           H
C                             =THERMAL CONDUCTIVITY OF THE MINERAL-SALT
C                              WATER SYSTEM IN CAL/DEG C CM/SEC
C     SW-----------IS EITHER CONST OR 0 (NOTE:IS A FUNCTION OF
C                  TEMPERATURE) WHERE
C                  GW*SW=THE HEAT SUPPLY RATE IN THE SEA WATER
C                  ONLY IN A UNIT VOLUME OF THE MIXTURE.
C     SS-----------IS A CONST (NOTE:IS A FUNCTION OF TEMPERATURE)
C                  WHERE
C                  GS*SS=THE HEAT SUPPLY RATE IN THE MINERAL ONLY
C                  IN A UNIT VOLUME OF THE MIXTURE.
C     EM-----------SAME AS OM
C     TM1----------INTIAL TEMPERATURE AT ALL POINTS
C
C
C     DECLARATION
C
        CHARACTER CHAR1*40
        CHARACTER PRINTOUT*8, TEMPERATURES*8
      INTEGER  STEP, TMAX, NUMBER OF STEPS, INCSTEP, OUTPUT COUNT
      REAL   PORPTM(105), POROS(105),DXX(105,3),O(105,3), P INIT
      REAL   K, NO,  TEMP(105), PRVPOR(105)
      REAL     SUM DPR, TOL DPR, TMPPTM(105)
      REAL SUMMARY(9,0:2000)
      INTEGER ISUM(0:6), OUTPUT INCREMENT
C
C     READ THE ABOVE VARIABLES OR CONSTANTS
C     (ASSIGN VALUES FOR THEM), AND PRINT THEIR VALUES.
C
        WRITE(6,*)'INPUT SOIL? [ILLITE OR OTHER] AT [20 C OR OTHER]
        READ(5,10) CHAR1
 10     FORMAT(A30)
        WRITE(12,15)CHAR1
 15     FORMAT(1X,A30)
        INCSTEP = 1
        WRITE(6,*)' PRINTED OUTPUT ? [All,Partial,or NL]  '
        READ(5,18,END=900) PRINTOUT
        WRITE(6,*)' OUTPUT TEMPERATURES ? [Y, N OR NL] '
        READ(5,18) TEMPERATURES
18      FORMAT(A8)
        OPEN(UNIT=1, FILE='@DATA', IOINTENT='INPUT', STATUS='OLD' )
C       AW=N**E=CONST, BELONGS TO THE SET OF [0.7,0.8]
        AW = 0.7
      READ(1,*)A,B,C,D,E,N,DT,GS,GW,GO,TOL,H CM,TMAX,OM,PO,NL, P INIT,NP
     1,BU,CVW,CVS,GG1,GG2,GG3,SW,SS,EM,TM1
        IF( PRINTOUT(1:1).EQ.'A' .OR. PRINTOUT(1:1) .EQ. 'P')
     W  WRITE(12,20)A,B,C,D,E,N,DT
      WRITE(6,20)A,B,C,D,E,N,DT
        IF( PRINTOUT(1:1).EQ.'A' .OR. PRINTOUT(1:1) .EQ. 'P')
     W  WRITE(12,22) GS,GW,GO,TOL,H CM,TMAX,OM, PO,NL, P INIT,NP
        WRITE(6,22) GS,GW,GO,TOL,H CM,TMAX,OM, PO,NL, P INIT,NP
 20   FORMAT('   THE FOLLOWING ARE INPUT DATA  ',/,' A= ',F14.9,
     1    ' B= ',F14.9,   ' C= ',E14.7,   ' D= ',F14.9,/,' E= ',F9.4,
     2    ' N= ',I6,   ' DT= ',F12.1)
 22   FORMAT(  ' GS= ', F3.1,    ' GW= ', F7.4,
     1 ' GO= ', E10.2,/, ' TOL= ',E9.2,   ' H CM= ',F9.6, ' TMAX= ',I5,
     2 /,' OM= ', F7.4, ' PO= ',F10.5,
     3 ' NL= ',I4,/,' P INIT= ',F6.4, ' NP= ',I2,/)
        IF( PRINTOUT(1:1).EQ.'A' .OR. PRINTOUT(1:1) .EQ. 'P')
     W  WRITE(12,25)BU,CVW,CVS,GG1,GG2,GG3,SW,SS,EM,TM1
      WRITE(6,25)BU,CVW,CVS,GG1,GG2,GG3,SW,SS,EM,TM1
 25   FORMAT(' BU= ',F5.1, ' CVW= ',F5.1,' CVS= ',F5.1,' GG1=',F10.6,
     1' GG2= ',F10.6,   ' GG3= ',F10.6,/,' SW= ',F5.2,
```

```
     2' SS= ',F5.2,/,' EM= ',F5.2,  ' TM1= ',F7.4,/)
      READ(1,*,END=900) MAX ITR, TOL DPR, NUMBER OF STEPS
      IF( NUMBER OF STEPS .LE. 0 ) NUMBER OF STEPS = 1000
      IF( MAXITR .LT. 2 ) MAX ITR = 2
      MAX USE = MAXITR * 2
      IF( TOL DPR .LE. 1.E-5 ) TOL DPR = 1.E-5
      NMS1=N-1
      NMS2=N-2
C
C
C     P=F    / (UNIT AREA) = F    = A*N**B   BY EXPERIMENT
C        S                   S
C
      P=PO
C
C
C
      ISUM(0)=1
      DO 30 I=1,5
        ISUM(I)=(I*N+1)/6
  30    CONTINUE
      ISUM(6)=N
        OUTPUT COUNT = 0
      STEP = 1
C
C
C     ASSIGN THE VALUES FOR VECTOR OF POROSITIES & TEMPERATURES
C     (INITIAL POROSITY & TEMPERATURE AT ALL POINTS)
C     AT TIME T.
C
      DO 40 I = 1, N
        PRVPOR(I) = P INIT
        PORPTM(I)=P INIT
        POROS(I)=P INIT
        TMPPTM(I)=TM1
  40    TEMP(I)=TM1
C     INITIAL T
      T=0.0
      DT MIN = DT
      DT PREV=DT
C
C
C
C-----------------------------------------------------------------------
C     1). DEFINE DX=H CM/(N-1)
C     2). INITIALIZE DXX(J,2)=DX FOR J=1 TO N ,AND DA=DX,DB=DX.
C     3). H INCH =H CM/2.54 (NOTE: 1 INCH = 2.54 CM)
C
      DX=H CM / NMS1
      DO 50 J=1,N
  50  DXX(J,2)=DX
      DA=DX
      DB=DX
C-----------------------------------------------------------------------
C-----------------------------------------------------------------------
C
C
C-----------------------------------------------------------------------
C     CHANGE CM TO INCHES.
      H INCH = H CM / 2.54
      SUMMARY(1,0)=T
      SUMMARY(2,0)=H INCH
      DO 55 I=0,6
```

```
         SUMMARY(I+3,0)=POROS(ISUM(I))
  55     CONTINUE
      WRITE(6,810) T, H INCH
      WRITE(6,830) (POROS(I),I=1,N)
      IF( TEMPERATURES(1:1) .EQ. 'Y' ) WRITE(6,840) (TEMP(I),I=1,N)
      IF( PRINTOUT(1:1) .EQ. 'Y' ) THEN
        WRITE(12,810) T, H INCH
        WRITE(12,830) (POROS(I),I=1,N)
        IF(TEMPERATURES(1:1) .EQ. 'Y' ) WRITE(12,840) (TEMP(I),I=1,N)
        ENDIF
C
C-----------------------------------------------------------------------
C     P=A*N**B  IMPIES N=EXP((ALOG(P/A))/B)
C
  60  CONTINUE
      ITER = 0
  65  NO=EXP((ALOG(P/A))/B)
C
C-----------------------------------------------------------------------
C     PH*A*RW=WEIGHT, WHERE RW IS THE UNIT WEIGHT OF WATER
C     IMPLIES P=WEIGHT/A=PH*RW
C     IMPLIES PH=P/RW=P/(GO*GW)    NOTE:RW IS GO*GW
      PH=P/(GO*GW)
C-----------------------------------------------------------------------
C
C
C     G *N     -G*N    +G N      =-G
C      2  I-1      I     1 I+1      3
C
C     IMPLIES -(G  /G)*N     +(G/G)*N  -(G  /G)*N      =(G  /G)
C                2       I-1          I     1       I+1      3
C
C     NOTE: Q(I,1)=-(G  /G)
C                    2
C     Q(I,2)=1.
C     Q(I,3)=-(G /G)
C               1
C
C     U(I)=G   /G
C          3
C
C
C     SH=A* N**B/(GO*GW)
C     FOR EACH POROSITY N AT EACH INTERIOR POINT.
C
      Q(1,1)=0.
      Q(1,2)=1.
      Q(1,3)=0.
      POROS(1)=NO
      DO 125 I=2,NMS1
      SH=A*EXP(B*ALOG(PRVPOR(I)))/(GO*GW)
C-----------------------------------------------------------------------
C
C     PERMEABILITY PER UNIT AREA K=C*N**D (CM/SEC)
      K=C*EXP(D*ALOG(PRVPOR(I)))
C-----------------------------------------------------------------------
C     CALCULATE THE COEFFICIENTS QN,RN,SN OF THE LEFT-HAND SIDE OF
C     THE CONSOLIDATION EQUATION.
C
C     QN=-K*EXP(-(E+1)*ALOG(PRVPOR(I)))*(PH*E+(B-E)*SH)
C     RN1=D+PRVPOR(I)*(E-1)-2*E
C     RN2=B+D-2*E-1
C     RN3=D-2*E+PRVPOR(I)*E-PRVPOR(I)
```

```
C     RN=-K*EXP(-(E+2)*ALOG(PRVPOR(I)))*(PH*E*RN1+SH*(B*RN2-E*RN3))
      QN=-K*SH*B/(AW*PRVPOR(I))
      RN=-K*SH*(B*(B-1.)/(PRVPOR(I)*PRVPOR(I))+D/(B*B))/AW
      SN=-K*D/PRVPOR(I)
C
C-----------------------------------------------------------------------
C     REDEFINE DA=DXX(I,2), DB=DXX(I+1,2)
C     (DA=DB=DX (OLD VALUE) IF TIME T IS NOT INCREASED BY DT)
C
      DA=DXX(I,2)
      I1=I+1
      DB=DXX(I1,2)
      DAPDB=DA+DB
      DAMDB=DA*DB
      AMP=DA*DAPDB
      BMP=DB*DAPDB
      C2P=2./BMP
      C2C=-2./DAMDB
      C2M=2./AMP
      C1P=DA/BMP
      C1C=(DB-DA)/DAMDB
      C1M=-DB/AMP
C
C-----------------------------------------------------------------------
C     FROM CONSOLIDATION EQUATION WE HAVE
C     QN*( (N(I+1)-2N(I)+N(I-1))/(DZ*DZ))+
C     (RN*DNODZ+SN) ( (N(I+1)-N(I-1))/(2DZ))
C     =(N(I)-PORPTM(I)/DT
C     AND SINCE
C                    2
C                  ( D N/     2)=C2P*N      +C2C*N    +C2M*N
C                       DZ            I+1          I          I-1
C         AT Z
C             I
C
C         AND (DN/DZ) =C1P*N      +C1C*N    +C1M*N
C                          I+1          I          I-1
C              AT Z
C                  I
C      THE CONSOLIDATION EQUATION BECOMES
C     Q(I,3)N(I+1)+Q(I,2)N(I)+Q(I,1)N(I-1)=U(I)
C         (NOTE: U(I)=-PORPTM(I)/DT)
C     AND Q(I,J)=1,2,3 ARE DESCRIBED AS FOLLOWS:
C
C
C
      DNODZ=C1P*PRVPOR(I+1)+C1C*PRVPOR(I)+C1M*PRVPOR(I-1)
      TERM=RN*DNODZ+SN
      Q(I,3)=QN*C2P+TERM*C1P
      Q(I,2)=QN*C2C+TERM*C1C-1.0/DT
      Q(I,1)=QN*C2M+TERM*C1M
      POROS(I)=-PORPTM(I)/DT
  125 CONTINUE
      Q(N,1)=0.
      Q(N,2)=1.
      Q(N,3)=0.
      POROS(N)=NO
C
C     POROSITIES AT TIME T+DT
C
      CALL TRDIAG( 105, N, Q, POROS )
C
C-----------------------------------------------------------------------
C
```

```
C
C
      SUM DPR = 0.
      DO 190 I = 1,N
        SUM DPR = SUM DPR + ABS( POROS(I) - PRVPOR(I) )
        PRVPOR(I) = POROS(I)
 190  CONTINUE
      ITER=ITER + 1
      IF (SUM DPR .GT. TOL DPR .AND. ITER .LE. MAX USE) GO TO 65
C
C-----------------------------------------------------------------------
C
C     SOLVE TEMPERATURE RIGHT HERE
C
      IF( TEMPERATURES(1:1) .EQ. 'Y' ) THEN
      Q(1,1)=0.
      Q(1,2)=1.
      Q(1,3)=0.
      TEMP(1)=TM1
      BRATIO=-B/BU
      F1EXP=(EXP(-(1/B)*ALOG(10*A)))*(1+BRATIO)
        DO 590 I = 2, NMS1
      DA=DXX(I,2)
      I1=I+1
      DB=DXX(I1,2)
      DAPDB=DA+DB
      DAMDB=DA*DB
      AMP=DA*DAPDB
      BMP=DB*DAPDB
      C2P=2./BMP
      C2C=-2./DAMDB
      C2M=2./AMP
      C1P=DA/BMP
      C1C=(DB-DA)/DAMDB
      C1M=-DB/AMP
      F1=A*(EXP(B*ALOG(POROS(I))))*(1-F1EXP*EXP(BRATIO*ALOG(
     1POROS(I))))
      F2=POROS(I)*GW*CVW+(1-POROS(I))*GS*CVS
      F3=GG1+GG2*POROS(I)+GG3*POROS(I)*POROS(I)
      F4=POROS(I)*GW*SW+(1-POROS(I))*GS*SS
      A1 =F3/F2
      DNODT=(POROS(I)-PORPTM(I))/DT
      A2=(F1/F2)*DNODT+(F4/F2)
C
C     FROM ENERGY EQUATION WE OBTAIN
C
C     (DENOTE TEMPERATURE BY V)
C     A (C2P*V     +C2C*V  +C2M*V    )=-A +(V(I)-V0(I))/DT
C      1       I+1.       I        I-1     2
C     THEN WE HAVE
C     Q(I,3)*V     +Q(I,2)*V  +Q(I,1)*V    =-A  -V0(I)/DT
C              I+1            I          I-1     2
C     AND Q(I,J),J=1,2,3 ARE DESCRIBED AS FOLLOWS:
C
      Q(I,3)=A1*C2P
      Q(I,2)=A1*C2C-1.0/DT
      Q(I,1)=A1*C2M
      TEMP(I)=-A2-TMPPTM(I)/DT
 590  CONTINUE
      Q(N,1)=0.
      Q(N,2)=1.
      Q(N,3)=0.
      TEMP(N)=TM1
```

```
C
      CALL TRDIAG( 105, N,Q, TEMP )
C
        ENDIF
      IF( SUM DPR .LE. TOL DPR ) GO TO 700
          TTEMP = T + DT
          IF(PRINTOUT(1:1).EQ.'A') WRITE(12,620) SUM DPR, TTEMP
          WRITE(6,620) SUM DPR, TTEMP
 620      FORMAT('0DID NOT CONVERGE, SUM DPR = ',G12.5,' T= ',G12.5)
      IF( PRINTOUT(1:1) .EQ. 'A' ) THEN
        WRITE(12,830) (POROS(I),I=1,N)
        IF(TEMPERATURES(1:1).EQ.'Y') WRITE(12,840)(TEMP(I),I=1,N)
        ENDIF
      WRITE(6,830) (POROS(I),I=1,N)
      IF( TEMPERATURES(1:1) .EQ. 'Y' ) WRITE(6,840)(TEMP(I),I=1,N)
 700  CONTINUE
C
C     SMALL H SUB T = (BIG H SUB T) * (1-N SUB T)/(1 - N SUB (T+DT))
C     CALCULATE THE HEIGHT
      DO 710 J=2,N
      POR AVG=(POROS(J)+POROS(J-1))/2.
      DXX(J,1)=DXX(J,2)*(1.-P INIT)/(1.-POR AVG)
 710  CONTINUE
      H CM=0.
      DO 720 I=2,N
 720  H CM=H CM + DXX(I,1)
      H INCH=H CM/2.54
      T = T + DT
      MAX USE = MAXITR
C
C     DEFINE DPOROS(I) = POROS(I) - PORPTM(I)
C
      DT PREV = DT
C-----------------------------------------------------------------
C
C
C-----------------------------------------------------------------
C     PRINT OUT 1). THE TIME T, SAMPLE HEIGHT H INCH
C
C                 2). THE POROSITY DISTRIBUTION OBTAINED AT ANY TIME T,
C                 3). TEMPERATURES
C
C
C
C
C      IF STEP IS DIVISIABLE BY INCSTEP THEN PRINTOUT T, H INCH
C      IMPLIES DT IS MULPLIED BY INCSTEP
       IF( (STEP/INCSTEP)*INCSTEP .EQ. STEP ) THEN
         WRITE(6,810) T, H INCH
 810     FORMAT(/,' THE DURATION IS  T =',F10.1,2X,
     1   'THE HEIGHT IN INCHES IS ',F12.4)
         WRITE(6,830) (POROS(I),I=1,N)
 830     FORMAT(/,' SOLUTIONS FOR POROSITY ',/,(10F8.4))
         IF( TEMPERATURES(1:1) .EQ. 'Y' ) WRITE(6,840)(TEMP(I),I=1,N)
 840     FORMAT(/,' SOLUTIONS FOR TEMPERATURES',/,(10F8.2))
         IF( PRINTOUT(1:1) .EQ. 'A' ) THEN
           WRITE(12,810) T, H INCH
           WRITE(12,830) (POROS(I),I=1,N)
           IF(TEMPERATURES(1:1).EQ.'Y') WRITE(12,840)(TEMP(I),I=1,N)
           ENDIF
C-----------------------------------------------------------------
C
         ENDIF
       IF( (STEP/INCSTEP)*INCSTEP .EQ. STEP ) THEN
```

```
      WRITE(6,870) INCSTEP, DT
870   FORMAT(' NOW USING OUTPUT INCREMENT =',I4,' STEPS',
     F      '    DT =',F10.1)
      WRITE(6,*)' CONTROL D TO STOP OR INPUT THE OUTPUT INCREMENT '
      READ(5,*,END=900) INCSTEP
      IF( INCSTEP .LT. 0 ) THEN
        WRITE(6,*) ' INPUT THE NEW  DT '
        READ(5,*,END=900) DT
        IF( PRINTOUT(1:1) .EQ. 'A' .OR. PRINTOUT(1:1) .EQ. 'P')
     W  WRITE(12,875)DT
875           FORMAT(' DT WILL NOW BE ',F12.1 )
        ENDIF
      INCSTEP = ABS(INCSTEP)
      IF( INCSTEP .LT. 1 ) INCSTEP = 100
      IF( DT .LT. DTMIN ) DT = DTPREV
      STEP = 0
      ENDIF
    STEP = STEP + 1
    RATIO = DT / DTPREV
    DO  880  I = 1, N
      SAVE   = POROS(I)
      POROS(I) = SAVE + ( SAVE - PORPTM(I) )*RATIO
    PRVPOR(I)=POROS(I)
      PORPTM(I) = SAVE
    SAVE=TEMP(I)
    TEMP(I)=SAVE+(SAVE-TMPPTM(I))*RATIO
    TMPPTM(I)=SAVE
880 CONTINUE
    OUTPUT COUNT=OUTPUT COUNT +1
    IF (OUTPUT COUNT .GT. 2000)OUTPUT COUNT=2000
    SUMMARY(1,OUTPUT COUNT)=T
    SUMMARY(2,OUTPUT COUNT)=H INCH
    DO 885 I=0,6
       SUMMARY(I+3,OUTPUT COUNT)=POROS(ISUM(I))
885    CONTINUE
    IF( T .LT. TMAX .AND. STEP .LT. NUMBER OF STEPS ) GO TO 60
900 WRITE(12,910)(ISUM(I),I=0,6)
910 FORMAT(10X,'TIME',6X,'HT. INCH',7(2X,'POROS(',I3,')'))
      OUTPUT INCREMENT = 1
    DO 950 ISU=0, OUTPUT COUNT
       IF (ISU .NE. (ISU/OUTPUT INCREMENT)*OUTPUT INCREMENT) GO TO 950
       WRITE(12,930) (SUMMARY(I,ISU),I=1,9)
        IF( SUMMARY(1,ISU) .GT. 60. ) OUTPUT INCREMENT = 61/DT
        IF( SUMMARY(1,ISU) .GT. 120. ) OUTPUT INCREMENT = 121/DT
        IF( SUMMARY(1,ISU) .GT. 600. ) OUTPUT INCREMENT = 601/DT
930    FORMAT(1X,F14.0,F12.4,7F12.4)
950    CONTINUE
    STOP
    END
```

APPENDIX C

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