DRAFT

SOIL AND ROCK MINERAL CONTACT AREAS REVISITED

by

Louis J. Thompson
Civil Engineering Department
Texas A&M University

October 20, 1980

The total normal force on any plane or wavey surface through saturated soil, sediment, concrete or rock is composed of force in the mineral fabric or matrix and the force in the water. These forces may be conceptualized as products of stresses and areas if the total area is very small. Of course, the sum of the two areas must equal the total area. Therefore

$$F_T = F_m + F_w$$

$$\sigma a_t = \hat{\sigma} a_c + u a_w$$

$$\sigma = \hat{\sigma} \frac{a_c}{a_T} + u \frac{a_w}{a_T}$$

and $$\sigma = \hat{\sigma} A_c + u\, A_w$$

where F_T = total normal force acting on

a_T = area total

σ = total normal stress

F_m = force in the mineral matrix acting on

a_c = mineral contact area

$\hat{\sigma}$ = mineral normal stress

F_w = force in the water acting on the

a_w = water area

u = pore pressure in the water

A_c = ratio of the area of the mineral to the total area

A_w = ratio of the area of the water to the total area

Usually it has been assumed that the water area is nearly equal to the total area and that the water area ratio is 1. If this assumption is true, it means that the stress in the mineral matrix must become infinite if any force is to be transmitted through the mineral matrix because the contact area has been assumed to be zero. Therefore, it seems that this question of whether or not the area of water is equal to the total area is at least open to question. Since the question becomes highly important in the study of oil well blowouts, hydrofracture, drill pipe sticking, lost circulation and above all in the development of methods to estimate the magnitude of formation overpressure it was decided to try to measure these disputed areas by some definite laboratory test.

A total of over 70 complete consolidation tests where mineral sediments were uniaxially strained in a saturated but drained state with a maximum load of 10,000 psi and then unloaded have been performed. It is known that when load is added to the consolidometer the height of the sample decreases as water drains out and when load is removed from the consolidometer rebound is expected as water is drawn into the sample. When water is not flowing in or out of the sample with free drainage the force in the mineral fabric is equal to the total applied force. Therefore, it is reasoned that if the height of the sample can be kept constant, the force in the mineral fabric will remain constant. If the pore pressure is increased, its force must be balanced by an increase in the total external force if the height does not change. If the area of the water is equal to the total area then the pore pressure must be equal to the additional stress applied to the top of the sample. If the balancing stress is less than the pore pressure then the area of the water is less than the total area. The ratio of the additional stress to the pore pressure is the ratio of the area of the water to the total area. Of course, friction must be accounted for in these calculations.

At first, the scheme was tried in a consolidometer built to measure the coefficient of earth stress at rest, K_o, and the results were encouraging. However, this gave measurements of vertical areas and there was some doubt about the deflection of the stainless steel sleeve. This equipment is shown in Figures 1 and 2. Therefore, a radial draining consolidometer was constructed that did not have these possible disadvantages. This consolidometer is shown in Figure 3.

Sumation of forces in the vertical direction for the consolidometer yields:

$$\Delta P_c + P_c + F_c = F_{mc} + F_{wc}$$

where ΔP_c = change in the vertical applied load

P_c = initial applied load when the pore pressure was zero

F_c = friction force in the consolidometer

F_{mc} = force in mineral matrix of the consolidometer

F_{wc} = force in the water of the consolidometer

Summation of forces in the permeameter in the vertical direction yields:

$$P_p - F_p = F_{wp}$$

where P_p = applied load on the permeameter

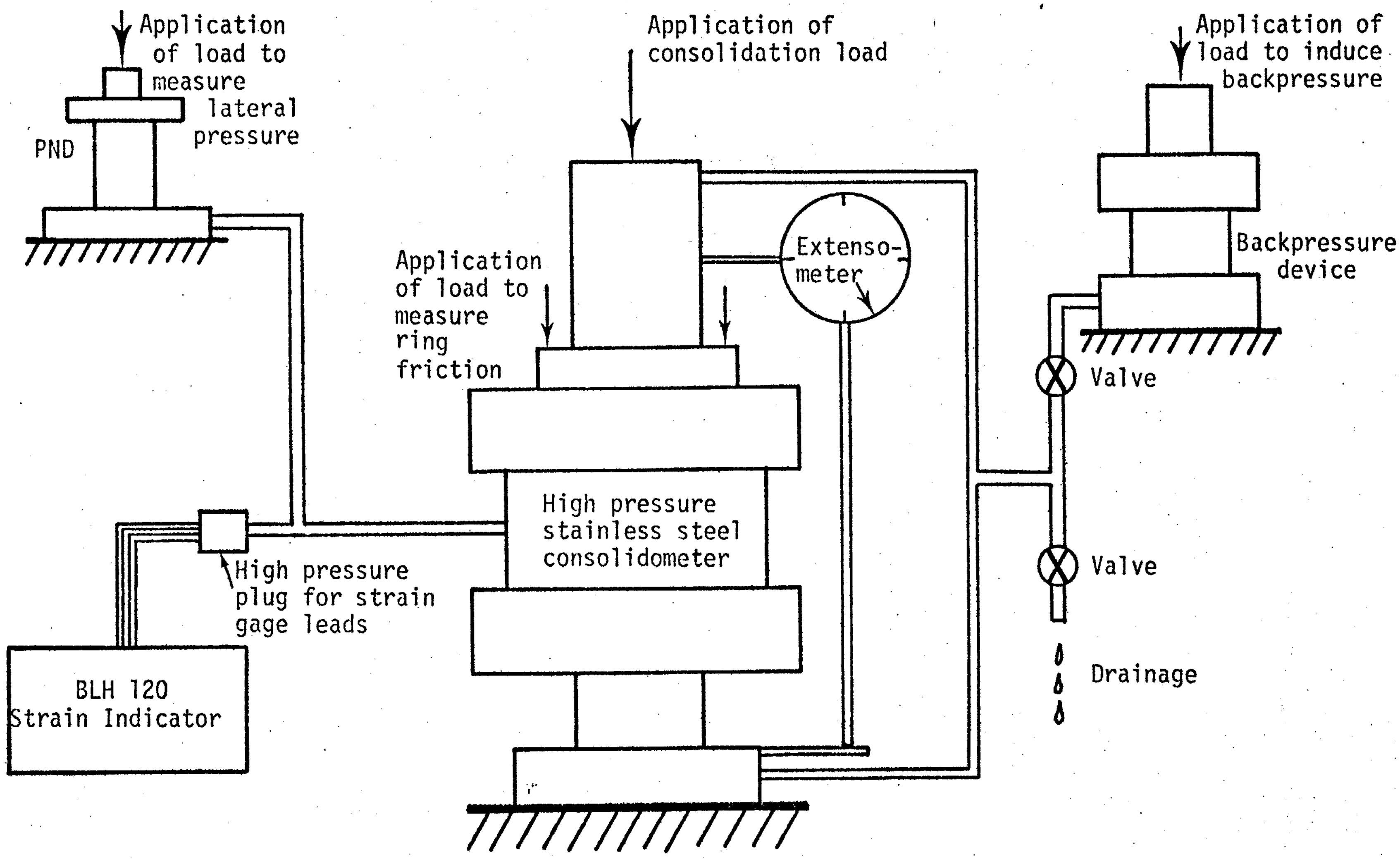

Fig. 1 - Schematic of High Pressure Consolidation System

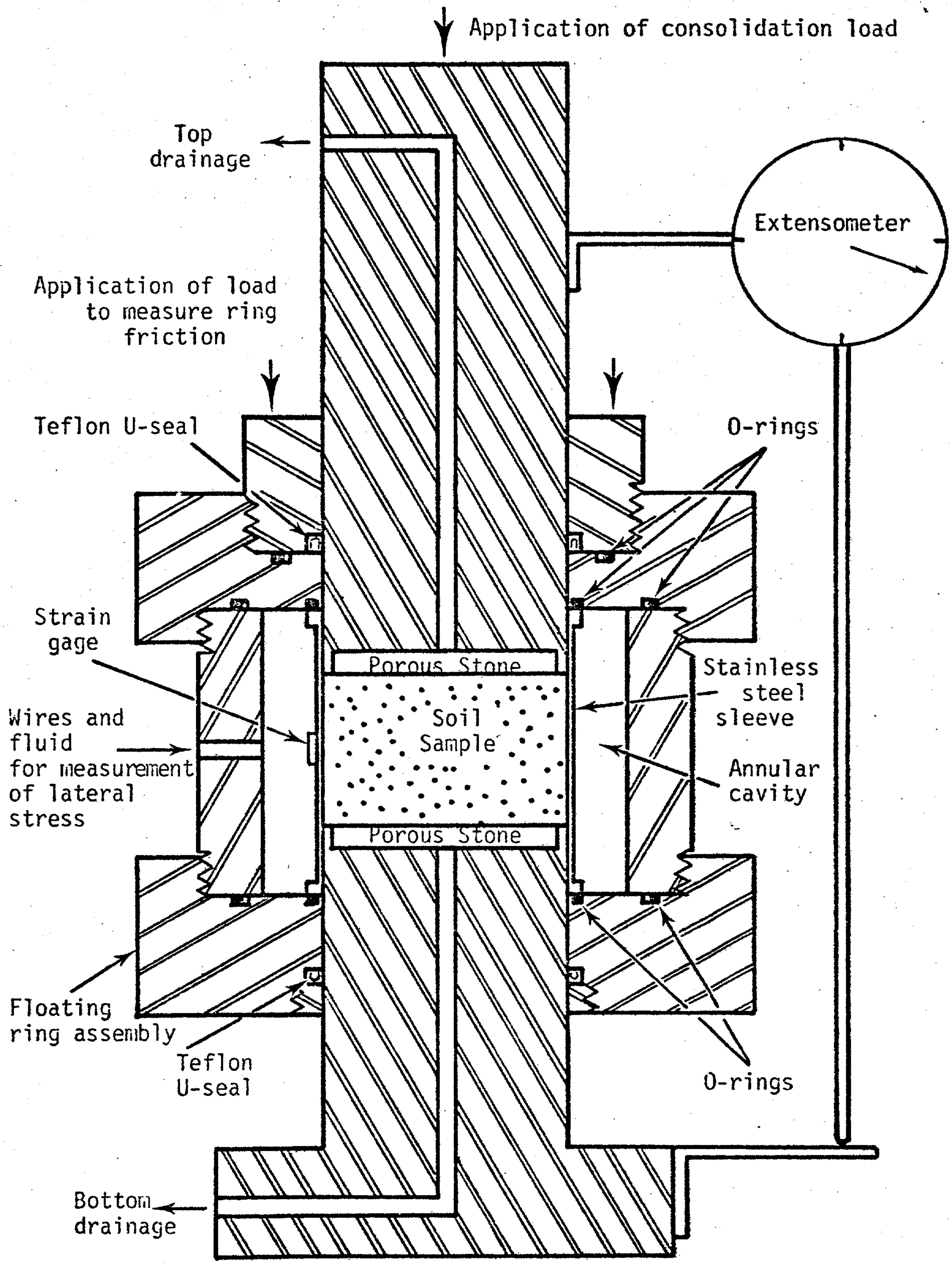

Fig. 2 - Cross-sectional Schematic View of High Pressure Stainless Steel Consolidometer

F_p = friction in the permeameter

F_{wp} = force in the water of the permeameter

Dividing these two equations by the respective areas of the pistons yields:

$$\Delta p_c + p_c + f_c = \frac{\hat{\sigma} a_c}{a_T} + u \frac{a_w}{a_T} = \hat{\sigma} A_c + u_c A_w$$

where Δp_c = change into vertical stress

p_c = initial vertical stress when the pore pressure is zero

f_c = the equivalent frictional stress in the consolidometer

$\hat{\sigma}$ = the stress in the mineral matrix

A_w = ratio of area of the water to the total area

A_c = ratio of the mineral contact area to the total area

u_c = the pore pressure in the consolidometer

Also $p_p - f_p = u_p$

where p_p = applied stress on the permeameter

f_p = the equivalent frictional stress in the permeameter

u_p = the pore pressure in the permeameter

Since the height of the consolidation specimen is kept constant by changing the load on the consolidometer as the permeameter is loaded, the stress p_c over the whole area of the consolidometer is equal to the stress in the soil $\hat{\sigma}$ times the contact area ratio A_c. This is accomplished by pouring #2 lead shot into buckets hung on the loading arms of the two lever systems of the consolidometer and the permeameter. The dial indicator hand of the extensometer is never allowed to move more than a very small part of one division which corresponds to 0.0001 inches of height change. Since

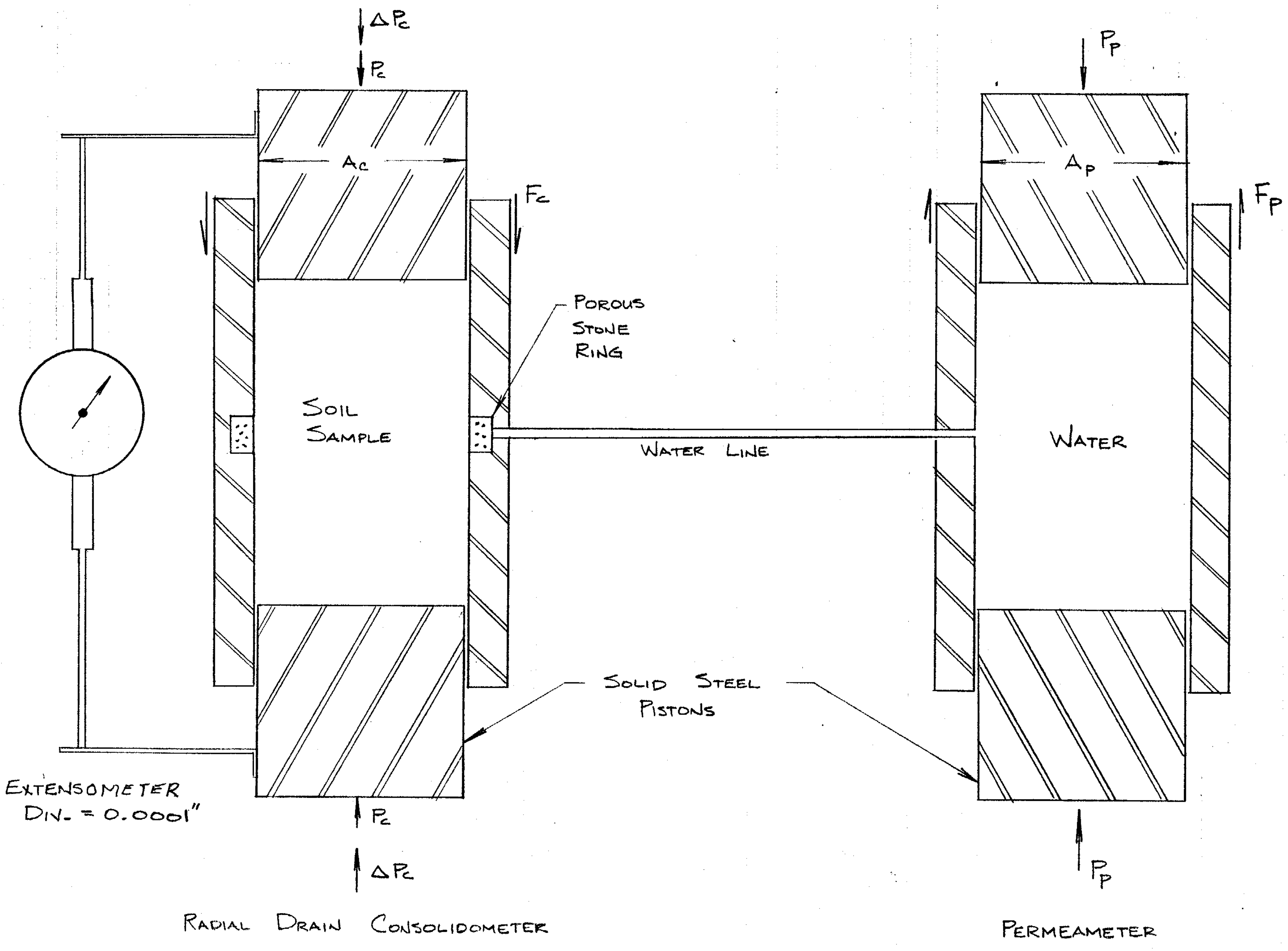

FIGURE 3
EXPERIMENTAL SCHEME TO MEASURE WATER TO TOTAL AREA ON HORIZONTAL PLANE IN SOIL SAMPLE.

$$p_c = \hat{\sigma} A_c$$

then $\Delta p_c + f_c = u_c A_w$ for the consolidometer

$p_p - f_p = u_p$ for the permeameter

The pore pressure u_c must equal u_p. Therefore

$$\Delta p_c + f_c = (p_p - f_p)\, A_w \qquad \text{Equation A}$$

When the tests are run there is a value of $p_p = p_{po}$ that just starts to cause motion when Δp_c is zero; therefore

$$f_c = (p_{po} - f_p)\, A_w$$

Substituting this value of the friction for the consolidometer back into the Equation A yields:

$$\Delta p_c + (p_{po} - f_p) A_w = p_p\, A_w - f_p\, A_w$$

this yields:

$$\Delta p_c = (p_p - p_{po})\, A_w$$

$$A_w = \frac{\Delta p_c}{p_p - p_{po}}$$

In Figure 4 the data for a test on Ottawa sand shows that for $p_c = 5000$ psi

$p_{po} = 1300$ psi

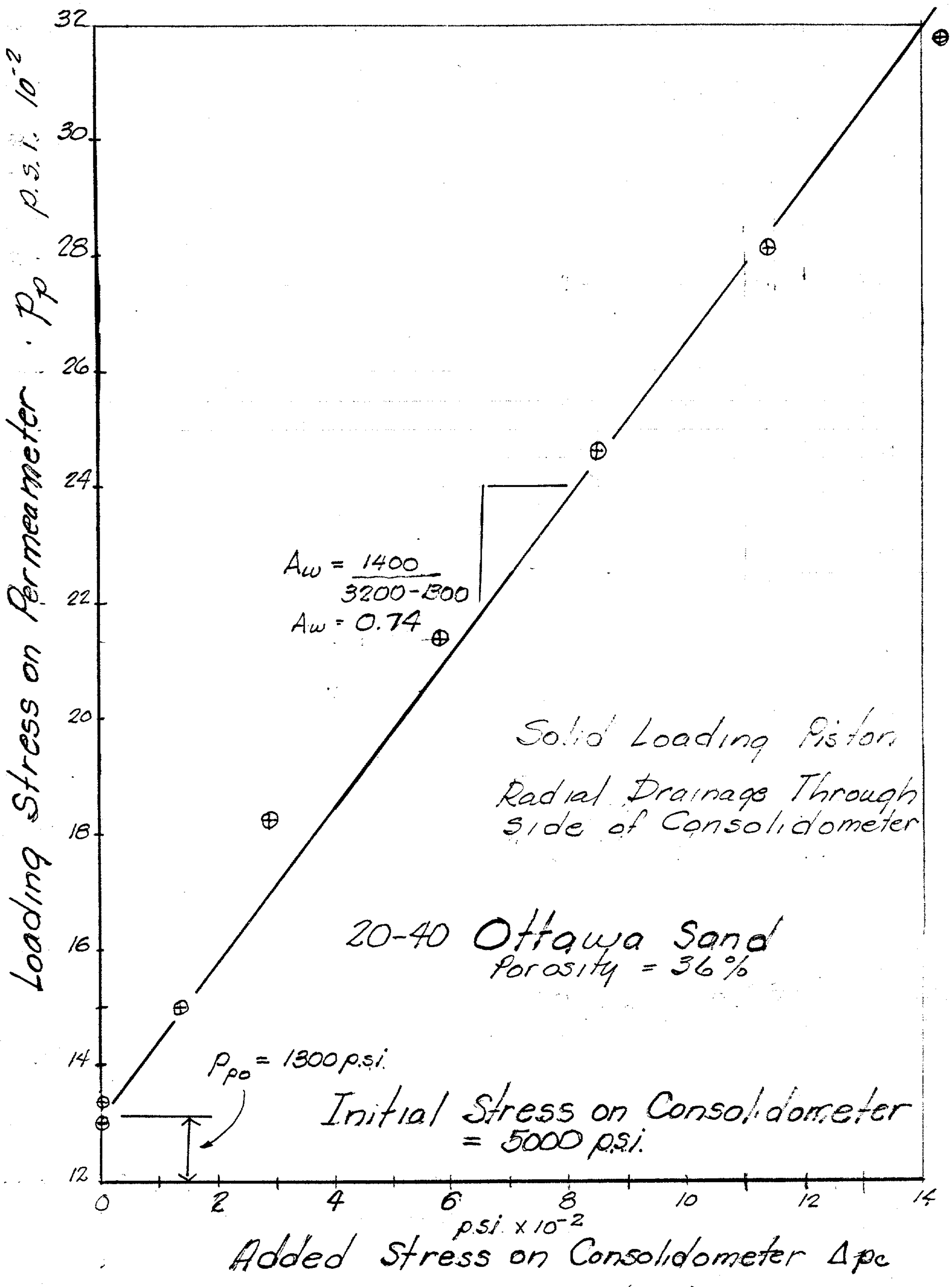

FIGURE 4 - Constant Height Consolidation Test - Radial Drainage

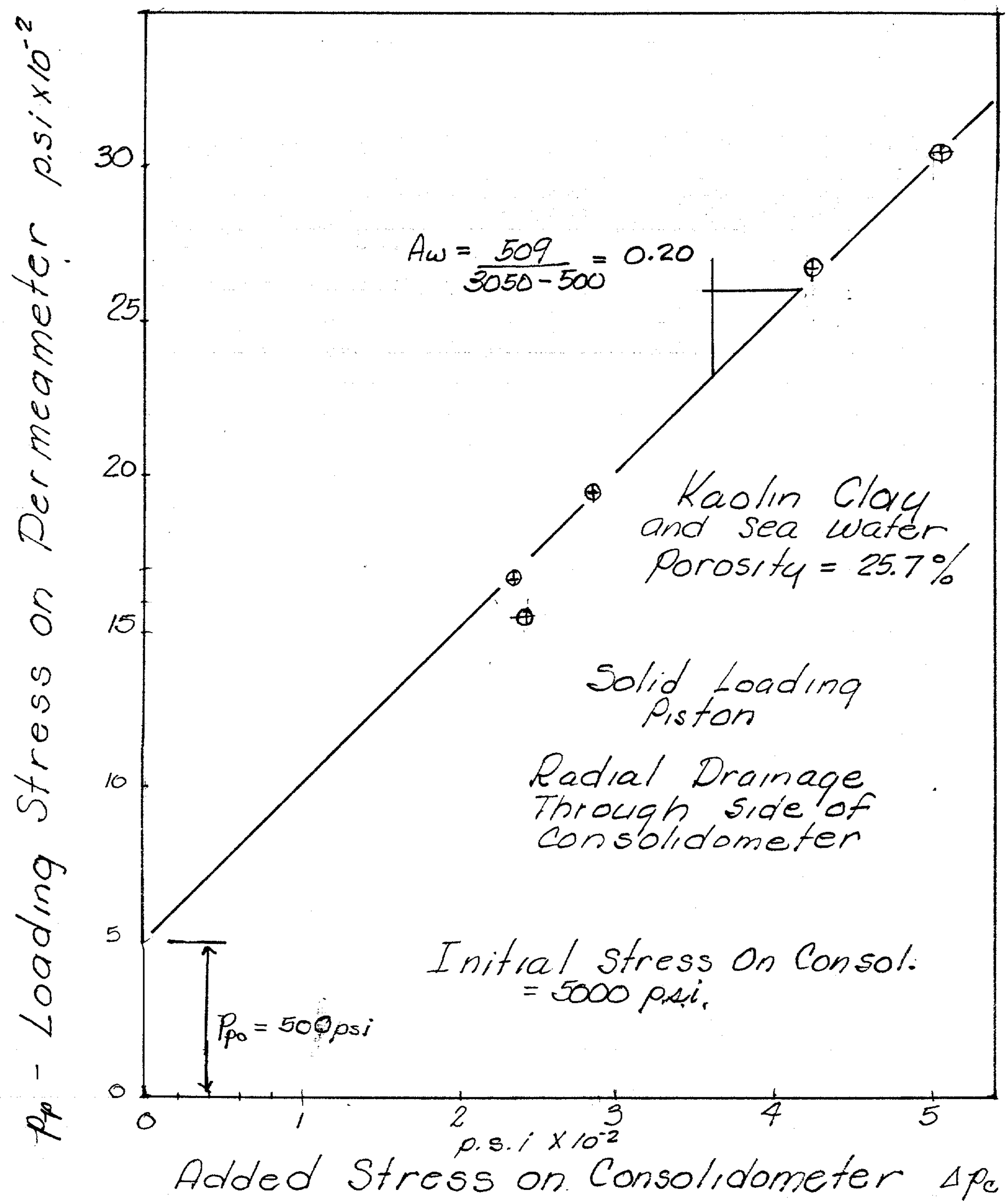

FIGURE 5 - Constant Height Consolidation Test - Radial Drainage

and when $p_p = 3200$ psi

$\Delta p_c = 1400$ psi

Therefore for the Ottawa sand with a porosity of about 36% the horizontal water area ratio is

$$A_w = \frac{1400}{3200-1300} = 0.74$$

In Figure 5 the data for a test on Kaolin clay shows that for $p_c = 5000$ psi when the porosity ratio is 0.257

$p_{po} = 500$ psi

and when $p_p = 3050$ psi

$\Delta p_c = 509$ psi

Therefore for the Kaolin clay with a porosity of 25.7% the horizontal water area ratio is

$$A_w = \frac{509}{3050-500} = 0.20$$

As mentioned, the original water area ratio tests were conducted in the device designed to measure the coefficient of lateral earth stress at rest (K_o). This work is described in the thesis by Callanan (1). Figures 1 & 2 show the schematic and cross sectional drawing for this K_o consolidometer.

A soil sample was placed in the consolidometer and slowly loaded so that drainage occurred. The thin stainless steel sleeve that enclosed the sample was confined by water inside a thick steel jacket. The pressure in this water was controlled so that the circumferential strain in the steel sleeve could be nulled. Strain gages were used to measure this small circumferential strain. In this scheme the load on the specimen, the pore pressure in the specimen and the null water pressure could all be controlled by independent lever systems.

After consolidation, under one load was finished and the pore pressure was zero and the null pressure to prevent lateral strain was known, a pore pressure was introduced into the specimen. This caused the null pressure to change. However, the pore pressure was always greater than the null pressure required to balance the internal water force. Following the same reasoning as before

$$A_T = \frac{\sigma_h - \sigma_{ho}}{u}$$

where σ_h = null water pressure when the pore water pressure was zero

σ_{ho} = null water pressure when the pore water pressure was u

u = induced water pressure inside the sample

Two different soils were tested at different porosities. The soil OS-3 was Ottawa sand. Table No. 1 gives the measured data for vertical stresses of 407 psi to 10,183 psi. The porosity ranged from 38.5% to 34.5%. The water area ratio along a vertical plane ranged from 0.713 to 0.784.

Table No. 2 gives the measure data for an undisturbed marine clay sample taken in Angola Basin off the west coast of Africa in 15,000 ft of water 700 ft below the mud line. The mineralogy is not available. The Atterberg limits are as follows: LL 127.5% and PI 53.8%. The initial porosity was 74.2%.

Table 2 gives the measured data for vertical stresses of 407 psi to 10,183 psi. The porosity ranged between 67.5% and 35.6%. The value of

TABLE 1 - Horizontal Stress Measurements with Pore Pressures Induced for Sample No. OS-3

σ_v Applied, psi	n, %	u Induced, psi	σ_h Measured, psi	A_w	Correlation Coefficient
407	38.5	0	182	0.713	0.98751
		45	199		
		90	242		
		134	273		
		179	304		
1385	38.1	0	645	0.723	0.99928
		90	701		
		179	770		
		269	840		
		359	900		
		448	965		
2688	37.6	0	1274	0.723	0.99893
		179	1385		
		359	1525		
		538	1657		
		717	1786		
5295	36.6	0	2584	0.784	0.99974
		179	2718		
		359	2861		
		538	3003		
		717	3137		
		896	3288		

TABLE 1 - (Continued)

σ_v Applied, psi	n, %	u Induced, psi	σ_h Measured, psi	A_w	Correlation Coefficient
10,183	34.5	0	5744	0.749	0.99911
		179	5875		
		359	6011		
		538	6145		
		717	6306		
		896	6412		
		1075	6546		
		1255	6680		

TABLE 2 - Horizontal Stress Measurements with Pore Pressures Induced for Sample No. MC-II-U

σ_v Applied, psi	n, %	u Induced, psi	σ_h Measured, psi	A_w	Correlation Coefficient
407	67.6	0	107	0.591	0.96233
		90	136		
		134	171		
		179	203		
		224	238		
1385	55.0	0	620	0.644	0.99995
		179	734		
		359	851		
		538	964		
		717	1083		
2688	48.5	0	1264	0.728	0.99963
		359	1509		
		717	1760		
		1075	2029		
		1434	2299		
		1792	2562		

TABLE 2 - (Continued)

σ_v Applied, psi	n, %	u Induced, psi	σ_h Measured, psi	A_w	Correlation Coefficient
5295	42.0	0	2440	0.740	0.99821
		359	2659		
		717	2921		
		1075	3240		
		1434	3538		
		1792	3787		
		2151	4024		
		2509	4273		
		2878	4520		
10,183	35.6	0	4942	0.716	0.99947
		359	5206		
		717	5473		
		1075	5704		
		1434	5960		
		1792	6239		

water area ratio ranged between 0.591 and 0.740.

These data from both Tables 1 and 2 are plotted in Figure 7. It shows that as the clay porosity decreased the vertical water area ratio increased. This was contrary to expectations. The sand sample water area ratio was not much influenced by the loading because the porosity changed so little.

No explanation is given for the seemingly anomalous data for the clay, although several have suggested that it has to do with the particle rotation as the porosity decreased.

The vertical and horizontal area ratios for the sand are much the same; 0.74 = A_{wH} and 0.713 to 0.785 = A_{wV}. For the Kaolin clay the horizontal area ratio A_{wH} = 0.20 and the vertical area ratio for the marine clay was 0.591 to 0.728. Therefore the anisotropy of clay may be as much as 3.5 to 4. A similar anisotropy has also been found for heat conductivity and permeability.

It has been suggested that the areas of water and contact area are important. It has also been shown in previous work that the force in the mineral may be represented by a power law function of porosity (2)(4). Therefore $\hat{\sigma}_c A_c = An^B$ for loading only and equilibrium requires that

$$\sigma_V = \hat{\sigma}_c A_c + A_w u$$

then $$\sigma_V = An^B + A_w u \qquad \text{Equation B}$$

for the uniaxial state of strain.

It is this last Equation B that may make it possible to estimate the pore pressure in a sand formation while drilling in an overlying shale formation. The pore pressure in the sand is generally thought to be less

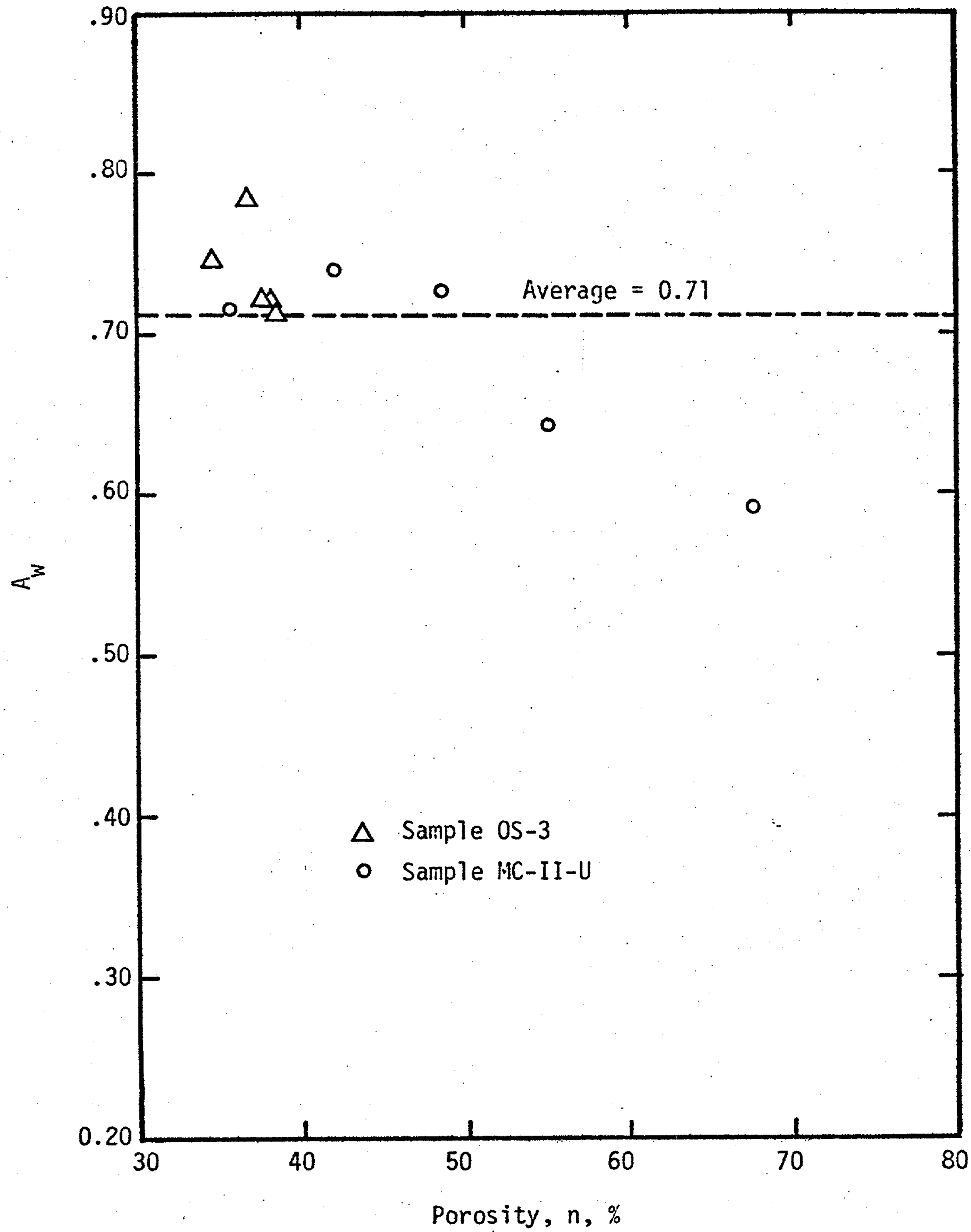

Fig. 7 - Variation of Water Area with Porosity as Measured for Two Soils

than the pore pressure in the shale because the compressible shale is the source of the pressure that causes drainage into the sand. The value of σ_V can be calculated from the weight of the overburden. If the material descriptors A_w, A and B can be determined from the type of minerals that make up the shale and if the porosity can be estimated from measurements on the cutting or from well logs, the only remaining variable in the last equation is the pore pressure u.

It was previously thought (4) that the A_w could be represented by a power law

$$A_w = n^E$$

This may not be true. Probably with more data it will be found that A_w is at least a 2nd order tensor quantity just as the permeability has been shown to be (3).

It should be mentioned that these areas measured are not for the failure conditions. They are for uniaxial strain.

REFERENCES

1. Callanan, Michael J., "The Prediction of Hydrofracture Pressure and K_0 During Drilling," thesis presented to Texas A&M University, College Station, Texas, in 1980, in partial fulfillment of the requirements for the degree of Master of Science.

2. Lee, Honwoo Thomas, "Compressibility and Permeability of Clays at High Pressure," thesis presented to Texas A&M University, College Station, Texas, in 1980, in partial fulfillment of the requirements for the degree of Master of Science.

3. Scheidegger, Adrian E., The Physics of Flow Through Porous Media, University of Toronto Press, Toronto, Canada, 1974.

4. Thompson, Louis J., "Overpressured Marine Sediments: The Thermomechanics of Progressive Burial," Final Report for Phase II, U.S.G.S. Grant No. 14-08-001-G-444, Texas A&M Research Foundation, College Station, Texas, July 1979.