THE SIGNIFICANCE OF DYNAMIC RESPONSE IN THE ESTIMATION OF FATIGUE LIFE

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ABSTRACT

The fatigue life of offshore structures is investigated under the conditions that dynamic response to waves is assumed to play a significant role.

The study emphasizes the variation of fatigue life as a function of the structural natural frequencies, the amount of modal damping and the extent of directional wave spreading.

The results may be used to assess the confidence bounds on fatigue life estimates that result from uncertainties in design stage estimates of structural natural frequencies, damping and wave spreading. The example of a single vertical cylindrical caisson is completed in detail. This example includes the explicit computation of wave radiation damping and hydrodynamic viscous damping.

A versatile single parameter wave spreading function is introduced and used to show the influence of wave spreading on fatigue for conditions varying from unidirectional to totally spread random seas.

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INTRODUCTION

The purpose of this analysis is to investigate the sensitivity of fatigue life calculations to various parameters which govern the response of an offshore structure to wave forces.

Such an analysis may be used to reveal the extent to which uncertainties in the estimates of such parameters will affect the estimated fatigue life.

This analysis does not consider the uncertainties in material properties or the fatigue damage accumulation models themselves. This area is left to the materials specialists. This study also leaves to others the analysis of the uncertainties associated with the description of the sea states to be encountered by the structure. Therefore, within a fixed, assumed framework of commonly accepted models of material behavior, damage accumulation formulas, and sea state description, the effect on fatigue resulting from the variation of three dynamic response parameters will be investigated. The three parameters are structural natural frequency, modal damping, and the directional spreading of the incident wave spectra.

Several example calculations are performed for the case of a simple vertical cantilever excited by waves. It is felt that the insight gained by these simple examples will be valuable in understanding the significance of these parameters when more complex structures are considered. The vertical cylinder was chosen because quantative results may be obtained without resorting to complicated numerical modeling.

VARIABLES WHICH INFLUENCE THE ESTIMATION OF STRUCTURAL RESPONSE

There are many variables which may introduce uncertainty into the response estimation procedure. Some of the more important ones are:

1. The model of the wave spectrum including directional spreading.
2. The estimation of structural natural frequencies.
3. The estimation and subsequent use of modal damping ratios.
4. The configuration of the real structure.
5. The structural model or idealization of the real structure.
6. The wave excitation model.
7. The computational methods used to estimate the response and eventually the member end forces and stresses.

In this analysis items four through seven will be fixed. In the example computations, a structural model and its idealization are selected and one method of wave force estimation will be used. The wave force model will assume that drag exciting forces are negligible and that finite wave amplitude effects are not significant. In any specific application the last two assumptions can and should be checked. However, for the computation of high cycle-low stress fatigue damage on large deepwater structures these assumptions are usually valid.

In the case of drag excitation the results of some recent research at MIT are mentioned. With these results the second order statistics of response may be estimated including nonlinear drag exciting forces.

The exclusion of finite wave amplitude effects is probably valid for large deepwater structures in low to moderate seas, which contribute the most to high cycle-low stress fatigue damage. The governing non-dimensional parameter is likely the ratio of wave amplitude to water depth for slender bottom mounted structures. However, this is an area in which some additional research is justified.

Item seven reflects the current discussion in the offshore industry regarding the best way to estimate the stress at a point in a structure with finite element methods, when dynamic response is significant. The most thorough investigation of the problem is that reported by Vugts and Hines [1].

They compared results obtained from finite element structural models, implemented in three different ways, as follows:

1. Direct solution of the complete dynamic equations of motion.
2. Normal mode superposition.
3. Full static, plus the dynamic contribution of a few modes considered to be important in the dynamic response.

Vugts and Hines concluded that direct solution was best and that pure normal mode superposition was unreliable by comparison. They also concluded that supplementing the dynamic contributions of the most significant normal modes with a full static solution was considerably better than pure normal mode superposition. They were not able to fully explain the discrepancies between the methods. It is for this reason that this issue is mentioned in this report. There exist some unresolved questions in the area of finite element dynamic modeling methods.

During the literature search conducted as part of this investigation an attempt was made to evaluate the likely sources of the discrepancies. Insufficient detail was available in the published literature to draw any precise conclusions. However, one common feature was evident. Farigue analyses as reported in the literature generally use some form of stress recovery, post processing computer routines. These are never fully described and it is nearly impossible to conclude if it was done properly. In the particular case of full static solutions supplemented by the dynamic response of a few modes it was possible that dynamic amplification factors were being applied to stress transfer functions which had been computed on the basis of the static deflection shape. This would clearly yield inaccurate results because the stresses depend on higher derivatives of the deflected shape. The mode shape of the lowest bending mode may appear very close to the static deflection shape, but the higher derivatives may be substantially different. Some additional work in this area seems justified.

For the example problems considered here the issue of which finite element procedure to use is avoided. This is possible because, for a simple cylinder, the quasi-static and dynamic contributions to the stress at the base of the cylinder may be estimated directly without the use of finite element programs. The same stress computation procedure is used throughout the sensitivity studies described here. Natural frequency, model damping, and wave spreading are the primary variables in this study. Resulting relative variations in computed fatigue life will not be particularly sensitive to the stress computation procedure.

THE FATIGUE ACCUMULATION MODEL

For the purpose of this study the assumed form of the fatigue damage accumulation model is that used by Crandall and Mark [2] when the stress history is assumed to be described by a narrow band random process. This formulation implicitly assumes a Palgren-Miner rule for damage acculumation. Equation 1 describes the mean rate of accumulation of the fatigue damage index for a location β in the structure due to a directionally spread random sea with mean direction θ_o.

$$F(\beta,\theta_o) = \frac{\nu_o^+}{c}\,(2^3\sigma_s^{\,2})^{b/2}\;\Gamma(1+b/2) \qquad (1)$$

$F(\beta,\theta_o)$ = the mean rate of accumulation of the fatigue damage index at position β, due to a wave field with nominal direction of propagation θ_o.

σ_s^2 = the mean square stress at position β.

ν_o^+ = the average zero upcrossing rate of the stress process in Hz.

$\Gamma(\ )$ = the Gamma function.

b,c = constants of the S-N fatigue curve of the material as defined by Equation (2), where N is the number of cycles to failure with a stress range S.

$$NS^b = c \qquad (2)$$

This model, and the material constants b and c are assumed fixed. This leaves ν_o^+ and σ_s^2 as variables to be considered.

ν_o^+ depends on the frequency content of the wave spectrum as well as the wave amplitude to stress transfer function for the structure. If the structure has no natural frequencies in the region of significant wave force, then the response is generally quasi-static in nature and ν_o^+ is governed primarily by the frequency content of the wave spectrum. When the stress is primarily due to the response at a natural frequency, then ν_o^+ is strongly dependent on the natural frequency which is governed by the structural specifications.

In both of the cases the response is approximately narrow band and the use of Equation (1) is appropriate. In the case that the response spectrum is composed of significant quasi-static and dynamic response peaks then it may be necessary to modify the above equation. One such modification is the use of a final correction factor, wuch as proposed by Wirsching [3], in which rain flow cycle counting procedures are used to obtain a correction factor to account for broad band stress spectra. The use of such a correction factor is assumed to be valid here.

The task is then to investigate the sensitivity of the mean square stress σ_s^2 and the average zero upcrossing frequency, ν_o^+, to variations in structural natural frequency, modal damping, and wave spreading.

QUASI-STATIC AND DYNAMIC CONTRIBUTIONS TO MEAN SQUARE STRESS

In this study, it is assumed that mean square stress at a point in a structure may be approximated by the sum of a quasi-static component due to low frequency waves and a dynamic component due to the damping controlled response of natural modes of the structure excited by the higher frequency components of the wave spectrum. This is comparable to the procedure of supplementing a full static finite element solution with the dynamic contributions of the significantly responding natural modes.

In this analysis the response is assumed to be quasi-static up to within one half power bandwidth of the lowest natural frequency of the structure. Furthermore, the lowest natural frequency is not allowed to be less than the peak frequency of the wave spectrum. The computation of the mean square stress is then accomplished by summing the mean square static component with the dynamic contributions.

The quasi-static component of stress at a specific location is assumed uncorrelated with the dynamic components. However, for closely spaced natural frequencies, correlation between the stress components of two or more natural modes may have to be considered. The partitioning of static and dynamic contributions to the total stress is illustrated in Figure 1, a stress spectrum with a quasi-static stiffness controlled peak and one damping controlled resonant peak.

The quasi-static mean square stress, σ_q^2, is obtained by integrating the stress spectrum up to $\omega_c = \omega_1 (1-2\xi)$ where ω_1 is the lowest natural frequency and ξ is the modal damping ratio of that mode.

$$\sigma_q^2 = \int_0^{\omega_c} S_s(\omega)\, d\omega \tag{3}$$

where $S_s(\omega)$ is the stress spectrum.

For a complex structure σ_q^2 could be computed from a static finite element model. For the vertical cylinder example evaluated in this report σ_q^2 is computed from a simple static model of a cylinder loaded by a force computed using the Morison equation.

The calculation of the static mean square stress may include the influence of drag forces, in which an equivalent linearization procedure has been used or a more accurate non-linear wave force spectrum has been computed using the results of Dunwoody [4]. Drag forces are negligible in the examples of this report.

This static approximation does neglect any dynamic amplification at frequencies below the cut off.

The average zero upcrossing frequency of the static component of stress is computed from the zero and second order moments of the truncated spectrum

$$\omega_q^2 = \frac{\int_0^{\omega_c} \omega^2 S_s(\omega)\, d\omega}{\int_0^{\omega_c} S_s(\omega)\, d\omega} = \frac{1}{\sigma_q^2} \int_0^{\omega_c} \omega^2 S_s(\omega)\, d\omega \tag{4}$$

σ_q^2 and ω_q^2 for the example calculations are assumed to be provided for the purposes of the remaining discussions. However, the effect of wave spreading on the quasi-static and dynamic components of the mean square stress are accounted for analytically later in this report.

The dynamic or damping controlled contributions to the mean square stress are computed separately. The area under the stress spectrum as shown in Figure 1 for $\omega_1(1-2\xi) \leq \omega \leq \omega_1(1+2\xi)$ is defined as the mean square dynamic response for mode 1.

There may be more than one mode which has significant dynamic response. The dynamic contribution of each must be separately evaluated. In this report the mean square dynamic response of all significant modes will be computed using techniques described by Vandiver [5]. In this reference it is shown that the mean square dynamic response of an individual mode x is given by:

$$\sigma_x^2 = \frac{2.5\ C_x\ \rho_w g^3}{m_x\ \omega_x^5} \qquad S_\eta(\omega_x)\ \frac{R_r\ (\omega_x)}{R_T\ (\omega_x)} \tag{5}$$

where

σ_x^2 : mean square dynamic deflection of the x^{th} normal mode

m_x : modal mass

ω_x : natural frequency

$S_\eta(\omega_x)$: wave amplitude spectrum evaluated at ω_x

ρ_w : density of water

g : acceleration of gravity

$\frac{R_r(\omega_x)}{R_T(\omega_x)}$: ratio of the radiation (wave making) to total modal damping evaluated at ω_x

This result is valid for lightly damped modes excited by linear wave forces. The constant C_x depends on structural geometry and wave spreading and will be evaluated in detail later. Through knowledge of the mode shape and structural details the mean square stress at a specific loation can be related to σ_x^2.

If there is more than one mode contributing in a significant way to the dynamic response then the stress at any specific location in the structure will depend upon the superposition of stresses from each mode. If the natural frequencies of each responding mode are different, (at least so that their damping controlled peaks as defined in Figure 1 do not overlap), then the stresses contributed by each may be assumed to be uncorrelated and the total mean square stress will be the sum of the mean square stresses due to each individual mode. This is a consequence of the fact that waves and hence wave forces of different frequencies are uncorrelated. If two peaks overlap then the correlation between stress components must be included.

The mean zero upcrossing frequency for mode x is simply $\omega_x/2\pi$. The mean upcrossing frequency for the combined static and dynamic stress history may be computed as a weighted sum of the individual contributions as shown below for a system with a single dynamic component.

$$\nu_o^+ \ (H_z) = \frac{1}{2\pi}\left[\frac{\omega_q^2\sigma_q^{\ 2} + \omega_1^2\sigma_{d1}^{\ 2}}{\sigma_q^{\ 2} + \sigma_{d1}^{\ 2}}\right]^{\frac{1}{2}} \tag{6}$$

where $\omega_q^{\ 2}$ and $\sigma_q^{\ 2}$ reflect the static response and $\omega_1^{\ 2}$ and $\sigma_{d1}^{\ 2}$ are the natural frequency and mean square stress contributed by mode 1.

THE EFFECT OF NATURAL FREQUENCY ON FATIGUE

If the fundamental flexural natural period of a steel jacket structure was taken to be 3.5 seconds for the purpose of fatigue life computation, and the as-installed natural period turned out to be 4.0 seconds, how much would the estimated fatigue life be reduced? An estimate may be obtained in the following way. Recalling equation 1 and adding γ, a Wirsching type correction factor to account for broadbanded spectral effects yields

$$F = \gamma \frac{\nu_o^+}{c} (2^3 \sigma_s^2)^{b/2} \Gamma(1 + b/2) . \tag{7}$$

Assuming that wave spreading effects have been taken into consideration, then a variation in the estimated natural period of a mode will influence three parameters in the above equation: γ, ν_o^+ and σ_s^2. σ_s^2 will change because its dynamic component will change. This is because the wave spectrum is a rapidly changing function of frequency, and as can be seen in Equation 5, the mean square dynamic response is proportional to the wave spectrum divided by the natural frequency raised to the fifth power. ν_o^+ will change as can be seen in Equation 6 because it depends on the natural frequency as well as on the mean square dynamic stress; γ may change because the broadbandedness of the stress spectrum may change. If a prime notation is used to denote the result with a shifted natural frequency, then the ratio of fatigue damage between two cases may be expressed as:

$$\frac{F'}{F} = \left(\frac{\gamma'}{\gamma}\right)\left(\frac{\nu_o^{+'}}{\nu_o^+}\right)\left(\frac{\sigma_s^{2'}}{\sigma_s^2}\right)^{b/2} \tag{8}$$

The two extreme cases are simple to evaluate. The first is when the estimated and actual natural periods are so short that the dynamic component of σ_s^2 is negligible. This is true for most structures when the lowest natural frequency corresponds to a period of 2.5 seconds or less. In this case $F'/F = 1.0$.

The more interesting extreme is when σ_{d1}^2, the dynamic component of stress of a single natural mode is assumed to be much larger than the static component. This may not be physically realizeable, but provides a useful upper bound on the variation of fatigue with natural frequency. One way to estimate this is through the ratio of fatigue damage at two different natural frequencies.

$$\frac{F'}{F} = \left(\frac{\nu_o^{+'}}{\nu_o^{+}}\right)\left(\frac{\sigma'_{d1}}{\sigma_{d1}}\right)^b = \left(\frac{\omega_1'}{\omega_1}\right)\left(\frac{\sigma_{d1}'}{\sigma_{d1}}\right)^b \tag{9}$$

Because the process is narrow banded the Wirsching correctionfactor reduces to 1.0 for both cases, and the upcrossing frequency reduces to the natural frequency divided by 2π.

$$\nu_o^{+} = \frac{\omega_1}{2\pi} \tag{10}$$

The only remaining step is to evaluate the frequency dependence of σ_{d1}^2, the mean square stress from dynamic response of the mode. This is quite easy and may be estimated directly from Equation 5, with one minor modification. In normal mode formulations the product of the modal mass and the natural frequency squared is simply the modal stiffness.

$$M_1\omega_1^2 = K_1 \tag{11}$$

If the natural frequency varies because the modal stiffness is different than expected then the effect on mean square stress should be evaluated using Equation 5. However, if the modal mass varies, then the effect on mean square stress should be evaluated after substituting Equation 11 into Equation 5, as follows.

$$\sigma_1^2 = \frac{2.5C_1\rho_w g^3}{K_1\omega_1^3} S_\eta(\omega_1) \frac{R_r(\omega_1)}{R_T(\omega_1)} \tag{12}$$

If it is assumed for small variations in natural frequency that the ratio between mean square modal deflection and mean square stress remains constant then the frequency dependence of the mean square stress is the same as that for mean square deflection as given in Equations (5) or (12). This is essentially an assumption that the mode shape does not change, which is not true, but is adequate here for the purpose of a simple check on sensitivity to changes in natural frequency. Therefore stress and deflection may be related as shown.

$$\sigma_{d1}^{2} = A^{2}\sigma^{2}_{1} \tag{13}$$

If there is any substantial wave spreading, such as cosine squared, then C_1 is only weakly dependent on frequency and is assumed not to vary. Similarly the ratio of radiation to total damping is assumed constant in comparison to other sources of variation. Lumping all constant quantities into A^2 in Equation (13), two expressions for σ_{d1}^{2} result, depending on whether the source of change was mass or stiffness.

$$\sigma_{d1}^{2} = \frac{A^2}{M_1} \frac{S_\eta(\omega_1)}{\omega_1^{5}} \quad \left(\begin{array}{l}\text{stiffness}\\ \text{changes}\end{array}\right) \tag{14}$$

$$\sigma_{d1}^{2} = \frac{A^2}{K_1} \frac{S_\eta(\omega_1)}{\omega_1^{3}} \quad \left(\begin{array}{l}\text{mass}\\ \text{changes}\end{array}\right) \tag{15}$$

It remains only to evaluate the frequency dependence of the wave spectrum.

Krogstad [6] has presented evidence that wind driven wave spectra may be modeled at frequencies higher than the frequency of the peak in the wave spectrum as given below:

$$S_{max}(f) = 1.62 \times 10^{-3} \quad f^{-4.6} \quad m^{2}\text{-sec} \tag{16}$$

This is the upper bound curve for spectral values, but possesses the frequency dependence characteristic of the high frequency side of wind driven wave spectra.

Expressed as a function of ω Equation (16) takes the form

$$S_{max}(\omega) = \frac{1}{2\pi} \times 1.62 \times 10^{-3} \left(\frac{\omega}{2\pi}\right)^{-4.6} \tag{17}$$

Assuming all of the constants in this spectrum are absorbed into the constant A^2 in equations 14 or 15 yields

$$\sigma_{d1}{}^2 = \frac{A^2}{M_1} \frac{1}{\omega_1{}^{9.6}} \quad \text{stiffness changes} \tag{18}$$

$$\sigma_{d1}{}^2 = \frac{A^2}{K_1} \frac{1}{\omega_1{}^{7.6}} \quad \text{mass changes} \tag{19}$$

Substituting each of these expressions into Equation (9) and setting the slope, b, of the S-N curve equal to 4.1 for welded tubular joints yields

$$\frac{F'}{F} = \left(\frac{\omega_1'}{\omega_1}\right)^{-14.6} \quad \text{(mass changes)} \tag{20}$$

$$\frac{F'}{F} = \left(\frac{\omega_1'}{\omega_1}\right)^{-18.7} \quad \text{(stiffness changes)} \tag{21}$$

Therefore, if the natural frequency is 10% greater than predicted, then the fatigue life will be increased by a factor of 4.02 or 5.94 depending on the source of the error.

These examples were upper bound situations in which the quasi-static contributions to mean square stress were assumed small. In most cases of practical interest both contributions will be of importance and the sensitivity to natural frequency variation will not be so extreme.

THE EFFECT OF DAMPING ON FATIGUE

A variation in the estimated damping of a normal mode influences the mean square dynamic contribution to the total stress directly, and the average upcrossing frequency indirectly, because of its dependence on the mean square dynamic stress.

To place an upper bound on the significance of an error in the prediction of modal damping an analysis similar to the previous section may be performed. If only the dynamic component of a single mode is presumed to contribute to the total mean square stress, then proceeding as before leads immediately to the following conclusion:

$$\frac{F'}{F} = \left\{ \left(\frac{R_r(\omega_1)}{R_T(\omega_1)}\right)' \Bigg/ \left(\frac{R_r(\omega_1)}{R_T(\omega_1)}\right) \right\}^{b/2} \tag{22}$$

All terms involving frequency directly cancel out because the natural frequency does not change in the example.

The method of computing mean square dynamic stress used in this analysis is somewhat unconventional and not widely used in the industry. Therefore, to reflect conventional practice the same upper bound on the sensitivity of fatigue damage calculations to variations in estimated total damping may be expressed as follows:

$$\frac{F'}{F} = \left(\frac{\xi_T}{\xi_T'}\right)^{b/2} \tag{23}$$

when ξ_T and ξ_T' are the estimated and actual total modal damping ratios, which are commonly estimated in the range from 1% to 5%.

It is the position of the author that the uncertainty in estimating the ratio of the radiation to total damping is much less than the uncertainty in estimating the total modal damping itself. Furthermore the use of Equation (12) leads to estimates of mean square dynamic stress which are bounded because the ratio of radiation to total damping is at most 1.0. No such upper bound exists when conventional methods of computing dynamic response are used.

Furthermore, conventional methods of estimating response require independent estimates of the modal wave force spectrum and the total modal damping. This ignores the fact that the modal radiation damping and the linear modal wave force spectrum are proportional to one another [5]. Thus two sources of uncertainty enter the calculations where only one exists.

For the sake of example, suppose in either formulation the damping is underestimated by a factor of 2.0. This will lead to an overestimate of the fatigue life by a factor of

$$(2)^{b/2} = 4.14 \quad \text{for } b = 4.1 \tag{24}$$

for the extreme case of no static contribution to the stress.

THE EFFECT OF WAVE SPREADING ON FATIGUE ESTIMATES

An accurate description of the wave spectrum must model the directional distribution of the wave energy as well as its distribution in frequency. The directional wave amplitude spectrum may be defined as $S_\eta(\omega,\theta)$, where ω and θ are the wave frequency and incidence angle respectively. The directional spectrum and the point spectrum are related as follows:

$$S_\eta(\omega) = \int_o^{2\pi} S_\eta(\omega,\theta)\, d\theta \tag{25}$$

For linear wave forces and structures which respond in a linear fashion, it is possible to define a wave amplitude to stress amplitude transfer function for any location in the structure, as a function of wave frequency and incidence angle. This transfer function is defined here as $H_{\eta s}(\omega,\theta)$. The directional stress spectrum is then given by

$$S_s(\omega,\theta) = |H_{\eta s}(\omega,\theta)|^2 S_\eta(\omega,\theta) . \tag{26}$$

The dependence on θ may be eliminated by integration:

$$S_s(\omega) = \int_0^{2\pi} |H_{\eta s}(\omega,\theta)|^2 S_\eta(\omega,\theta)\, d\theta \tag{27}$$

The most common limiting form is the case of unidirectional waves from an angle θ_o. In this case,

$$S_\eta(\omega,\theta) = S_\eta(\omega)\ \delta(\theta-\theta_o) \tag{28}$$

and

$$S_s(\omega) = |H_{\eta s}(\omega,\theta_o)|^2 S_\eta(\omega) . \tag{29}$$

It is common practice in the offshore industry to assume that the wave spectrum is unidirectional. The stress spectrum at each point of interest is then computed for a discrete number of directions, varying from 1 to 16.

The annual stress history is then built up by weighting the discrete components on the basis of climate data. Such an approach generally overestimates the stress spectra and therefore underestimates the fatigue life of the structure.

For the purpose of comparison, the other limiting form of the directional wave spectrum is the case of totally diffuse seas, in which waves are equally probable from all directions.

In this case,

$$S_\eta(\omega,\theta) = \frac{1}{2\pi} S_\eta(\omega) \tag{30}$$

and

$$S_s(\omega) = S_\eta(\omega) \frac{1}{2\pi} \int_0^{2\pi} |H_{\eta s}(\omega,\theta)|^2 \, d\theta =$$

$$= S_\eta(\omega) < |H_{\eta s}(\omega,\theta)|^2 >_\theta \qquad (31)$$

The resulting stress spectrum is simply the product of the point wave amplitude spectrum and the mean square value of the stress transfer function with respect to incidence angle.

For linear wave forces,

$$|H_{\eta s}(\omega,\theta)| = |H_{\eta s}(\omega, \theta+\pi)| \qquad (32)$$

and therefore it is only necessary that the waves be uniformly distributed over π radians to achieve the result of Equation (31). Structural symmetry may reduce still further the total angle over which the wave spectrum must be spread to achieve the same conclusion. It is the position of the author that a realistic amount of spreading is sufficient in many cases to achieve the result of Equation (31). One such case is the example of the heave response of a tension leg platform in seas described by a cosine squared spreading function as presented in Reference [5]. The results for a single vertical cylinder are presented in this report.

If for a given structure it is found that the results of Equation (31) apply, then the fatigue life at any specific point becomes insensitive to annual variations in the mean direction of the sea, and depends only on the point wave spectra.

Wave spreading, structural natural frequencies, and modal damping ratios have been shown to be important parameters in the estimation of stress spectra and hence the mean square values of stress.

The application of the general results derived in this paper up to this point are best illustrated by an example calculation. The case of a single cantilevered vertical cylinder is presented in the next section.

THE EFFECT OF WAVE SPREADING ON THE FATIGUE OF A VERTICAL CYLINDER

The quantitative variation in estimated fatigue life, which results from variation in natural frequencies, damping or wave spreading may be clearly demonstrated with this simple structural model. Many of the calculations may be done in closed mathematical form, or obtained by simple numerical models. Such a complete analysis may be helpful in establishing a methodology which may be followed in the analysis of more complex structures. Furthermore, quantitative results obtained here will indicate the qualitative results to be expected in more complex structures.

The model structure is shown in Figure 2. It is a single, vertical cylinder with a concentrated mass, which for mathematical simplicity is located at the water line. The size of the concentrated mass is varied to adjust the natural periods of the lowest bending modes. The structure has a diameter, D, a wall thickness, t, and a water depth and cylinder length, h. For the purpose of computing the natural frequencies and modal mass the structure is assumed to have a uniform added mass coefficient of 1.0. That is, the fluid added mass is equal to the displaced volume. The x and y coordinates are used to describe the horizontal plane. The z coordinate is positive up from the bottom. Hence the water line is at $z = +h$. The mode shape of the lowest bending mode is defined as $\psi(z)$.

In an earlier section a linear transfer function between wave amplitude and stress was defined. For the vertical cylinder that transfer function takes on a simple form for the principal normal stress found at the base of the cylinder at an angle β measured from the x axis.

$$H_{\eta s}(\omega,\theta,\beta) = H_{\eta s}(\omega)\cos(\theta-\beta) \tag{33}$$

where $H_{\eta s}(\omega)$ is the wave amplitude to maximum principal stress transfer function at the base of the cylinder at the angle $\beta=\theta$. The coordinates β and θ are defined in Figure 3.

The $\cos(\beta-\theta)$ accounts for the variation of stress around the base of the cylinder for any location designated by the angle β, which may differ from the wave incidence angle θ. The stress spectrum at β is given by

$$S_s(\omega,\beta,\theta) = |H_{\eta s}(\omega)|^2\cos^2(\theta-\beta)\ S_\eta(\omega,\theta) \tag{34}$$

Assuming that the wave spectrum is separable into the product of the point spectrum and a spreading function as follows,

$$S_\eta(\omega,\theta) = S_\eta(\omega)\ D(\theta) \tag{35}$$

then the stress spectrum becomes

$$S_s(\omega,\beta,\theta) = |H_{\eta s}(\omega)|^2 S_\eta(\omega)\cos^2(\theta-\beta)\ D(\theta)\ . \tag{36}$$

If we let $\theta = \beta$ and consider the case of unidirectional waves from $\theta = 0$, then we find that

$$S_s(\omega,0,0) = |H_{\eta s}(\omega)|^2\ S_\eta(\omega)\ . \tag{37}$$

This quantity is the maximum stress spectrum at the base due to unidirectional waves.

Equation (36) may be integrated over frequency and angle to obtain the mean square stress for situations with a more realistic amount of spreading and for various locations β. This is expressed below:

$$\sigma_s^2(\beta) = (\sigma_{qo}^2 + \sigma_{do}^2)\int_o^{2\pi}\cos^2(\theta-\beta)\ D(\theta)\ d\theta \tag{38}$$

where

$$\sigma^2_{qo} = \int_{o}^{\omega_1(1-2\xi)} S_\eta(\omega) \, |H_{\eta s}(\omega)|^2 \, d\omega \quad , \qquad (39)$$

$$\sigma^2_{do} = \int_{\omega_1(1-2\xi)}^{\omega_1(1+2\xi)} S_\eta(\omega) \, |H_{\eta s}(\omega)|^2 \, d\omega \qquad (40)$$

where σ^2_{qo} and σ^2_{do} are the maximum static and dynamic mean square stress components due to unidirectional random waves.

The static component, σ^2_{qo}, may be computed by a simple computer program. The program computes the overturning moment on a stationary cylinder due to unidirectional random waves and then uses the simple relationship between bending moment and stress for slender beams.

$$\sigma^2_{qo} = \frac{\sigma^2_m \, D^2}{4I^2} \qquad (41)$$

where σ^2_m is the mean square static moment and 2I/D is the section modulus of the cylinder. The moment is computed using the distributed wave forces as predicted by Morison's equation.

The dynamic component, σ^2_{do}, of the maximum mean square stress may be estimated by considering the modal dynamic response predicted in Equation (5). First it is necessary to discuss briefly the behavior of a vibrating cylinder.

Solution of the eigenvalue problem reveals that the two lowest vibration modes have the same natural frequency and mode shape, but vibrate in planes that are orthogonal to one another. The orientation of these planes is not unique and may be defined to simplify solution of the problem. In this example they are assumed to correspond to the x and y axes, and are designated x and y by subscripts. Only these two lowest bending modes are considered to contribute significantly to the dynamic response. The response of the higher modes is ignored. σ_{do}^2 is the dynamic contribution to the total mean square stress for the case of unidirectional waves and $\beta = \theta_o$, the angle of incidence of the waves. σ_{do}^2 must be proportional to the mean square modal deflection given by Equation (5) for the same condition of spreading. Furthermore, in the case of unidirectional waves incident on the structure at $\theta = 0$, the only deflections will be in the x direction. The first bending mode in the x direction contributes all of the response. It remains to calculate the undetermined constant C_x. From Reference [5] C_x is given as

$$C_x = \frac{\int_o^{2\pi} S_\eta(\omega,\theta)\,|\Gamma_x(\omega,\theta)|^2 d\theta}{S_\eta(\omega)\ \frac{1}{2\pi}\int_o^{2\pi} |\Gamma_x(\omega,\theta)|^2\ d\theta} \tag{42}$$

where $|\Gamma_x(\omega,\theta)|$ is the modal wave force per unit wave amplitude for mode x and may in this case be expressed as separate functions of θ and ω :

$$|\Gamma_x(\omega,\theta)| \quad = \quad |\Gamma(\omega)\cos\theta| \tag{43}$$

Assuming the directional wave spectrum is as given in Equation (35), and using the result of Equation (43), C_x reduces to the following simple expression:

$$C_x \quad = \quad 2\int_o^{2\pi} \cos^2\theta\ D(\theta)d\theta \tag{44}$$

Similarly,

$$|\Gamma_y(\omega,\theta)| \quad = \quad |\Gamma(\omega)\ \sin\theta| \tag{45}$$

and

$$C_y = 2 \int_0^{2\pi} \sin^2\theta \; D(\theta) \; d\theta \tag{46}$$

For the case of unidirectional waves traveling in the direction $\theta = 0$, $D(\theta)$ becomes

$$D(\theta) = \delta(\theta) \tag{47}$$

and

$$C_x = 2.0$$

$$C_y = 0 \tag{48}$$

It follows from Equation (5) that the mean square displacement response σ_{xo}^2 is given by

$$\sigma_{xo}^2 = \frac{5\rho_w g^3}{M_x \omega^5} \; S_\eta(\omega_x) \; \frac{R_r(\omega_x)}{R_T(\omega_x)} \tag{49}$$

For each mode shape the stress and displacement are linearly related. Therefore a constant B^2 must exist which may be used to relate the mean square displacement due to unidirectional waves to the mean square stress as shown.

$$\sigma_{do}^2 = B^2 \sigma_{xo}^2 \tag{50}$$

and may be substituted into Equation (38) to yield

$$\sigma_s^2 = (\sigma_{qo}^2 + B^2 \sigma_{xo}^2) \int_0^{2\pi} D(\theta) \; \cos^2(\theta-\beta) \; d\theta \tag{51}$$

The effect of spreading and of stress point location angle on the total mean square stress are both contained in the integral. The above equation is now evaluated for three different spreading models.

Case 1. $D(\theta) = 1/2\pi$, omnidirectional spreading. After integrating

$$\sigma_s^2 = \frac{1}{2} (\sigma_{qo}^2 + B^2 \sigma_{xo}^2) \qquad (52)$$

This spreading is the most extreme and reduces the mean square stress at any angle β to 1/2 that for $\beta = 0$ and unidirectional waves at $\theta = 0$.

Case 2. $D(\theta) = \frac{2}{\pi} \cos^2(\theta-\theta_o)$ (53)

This is the so-called cosine squared spreading function. Arbitrarily requiring that the mean angle of incidence θ_o equals zero, and evaluating the integral yields

$$\sigma_s^2(\beta) = [\sigma_{qo}^2 + B^2\sigma_{xo}^2][3/4 \cos^2\beta + 1/4 \sin^2\beta] \qquad (54)$$

For the case that $\beta = 0$, cosine squared spreading reduces the total mean square stress to 75% of that in unidirectional waves.

Case 3. $D(\theta-\theta_o) = \dfrac{\sqrt{1-e^2}}{2\pi(1-e \cos(\theta-\theta_o))}$ (55)

In polar coordinates, $D(\theta-\theta_o)$ describes a family of ellipses based on the eccentricity parameter e. One of the focii of the ellipse lies on the origin of the coordinate system and the other focus lies along the direction θ_o. The eccentricity parameter can take on any value between zero and one. Zero corresponds to a completely diffuse sea with equal amplitudes of waves propagating in all directions. One corresponds to a unidirectional sea propagating in the direction θ_o. The spreading function, $D(\theta-\theta_o)$, is suitably normalized so that the point wave amplitude spectrum, computed by integrating the directional spectrum over all angles, equals the original point spectrum. This angular spreading function has been chosen over other possibilities because the amount of spreading is a smooth function of a single parameter. The parameter, e, can be used as the measure of spreading in the computation of fatigue resistance as a function of angular spread of the directional wave spectrum. Figure 4 shows the function $D(\theta)$ for various values of the parameter, e.

Again the problem is simplified if we require the mean angle of incidence θ_o to coincide with the x axis. The integral over angle in Equation (51) becomes

$$\int_0^{2\pi} \frac{\cos^2(\theta-\beta)\ \sqrt{1-e^2}\ d\theta}{2\pi(1-e\ \cos\theta)} \tag{56}$$

$$= \quad \cos^2\beta\ G + \sin^2\beta(1 - G) \tag{57}$$

$$\text{where} \quad G = \int_0^{2\pi} \frac{\cos^2\theta\sqrt{1-e^2}\ d\theta}{2\pi(1-e\ \cos\theta)} \tag{58}$$

The values for G corresponding to values of e ranging from 0 to 1.0 are shown in Table 1. For the particular case of $\beta = \theta_o = 0$ then

$$\sigma_s^2(0) \quad = \quad [\sigma_{qo}^2 + B^2\sigma_{xo}^2]G \tag{59}$$

Thus, e = .95 yields a value of G = .76 which is essentially the same as cosine squared spreading in the previous example.

It is useful to investigate how spreading affects the mean rate of fatigue damage accumulation, F, as defined in Equation (1). This shall be done here by computing the ratio of F for an arbitrary value of spreading, e, to that of F when the waves are unidirectional (e = 1.0).

$\sigma_s^2(\beta)$ for unidirectional waves is

$$\sigma_s^2(\beta) \quad = \quad [\sigma_{qo}^2 + B^2\sigma_{xo}^2]\ \cos^2\beta \tag{60}$$

Inserting this expression into the fatigue damage equation and doing the same with the results obtained by applying Equations (51), (55), (56), (57), and (58) and taking the ratio yields:

$$\frac{F(e,\beta)}{F(1,\beta)} = \left[\frac{\cos^2\beta\ G + \sin^2\beta(1-G)}{\cos^2\beta}\right]^{b/2} \qquad (61)$$

For the case of $\beta = 0$ this reduces to $G^{b/2}$. For $b = 4.1$, $G^{b/2}$ is also tabluated in Table 1 for various values of e.

For cosine squared spreading and $\beta = 0$, this ratio equals 0.55 and for complete spreading it is 0.24, as compared to 1.0 for the unidirectional case. In other words, for $\beta = 0$, cosine squared spreading reduces the fatigue damage to approximately one-half that for unidirectional waves.

TABLE 1

Spreading parameter e versus G and $G^{b/2}$ for $b/2 = 2.05$

e	G	$G^{b/2}$	Nature of wave spreading
0	0.5	.24	omnidirectional
.5	0.53	.27	
.7	0.58	.33	
.8	0.62	.38	
.85	0.65	.41	
.9	0.69	.47	
.95	0.76	.57	approx. cosine squared
.99	0.87	.75	
1.0	1.0	1.0	unidirectional

THE NATURAL FREQUENCIES AND MODE SHAPES OF THE CYLINDER

The first natural frequency and mode shape for a beam free at one end and fixed at the other are given by Biggs [7]:

$$\omega_1 = \frac{3.52}{\ell^2}\sqrt{\frac{EI}{m}} \tag{62}$$

where: E = Young's modulus

I = moment of inertia of the cross section

m = mass per unit length of the beam

ℓ = length of the beam

The mode shape is

$$\psi_1(z) = A\left[-\left(\frac{\cos a\ell + \cosh a\ell}{\sin a\ell + \sinh a\ell}\right)(\sinh az - \sin az) + \right.$$
$$\left. + \cosh az - \cos az\right] \tag{63}$$

where $a\ell = 1.875$.

A is an arbitrary constant which is adjusted in this example to make the value of the mode shape at the tip ($z = \ell$) equal to 1.0. For $\psi_1(z = \ell) = 1.0$, $A = 0.50$.

The bending moment per unit tip deflection in this mode shape may be computed from the moment curvature relation.

$$\frac{M(z)}{EI} = \frac{d^2\psi_1(z)}{dz^2} = a^2A\left[-.734(\sinh az + \sin az) + \right.$$
$$\left. + \cosh az + \cos az\right] \tag{64}$$

The maximum bending moment occurs at the base, $z = 0$.

$$\frac{M(0)}{EI} = 2a^2A = 2\left(\frac{1.875}{\ell}\right)^2 A \tag{65}$$

For a thin-walled tube the stress due to bending at a location on the circumference identified by the angle β, due to deflections of the tip in the direction designated θ, is given by

$$S(\beta,\theta,t) = \frac{M(0)R}{I} \cos(\theta-\beta)\, q(t) \tag{66}$$

$$= 2A\left(\frac{1.875}{\ell}\right)^2 E\, R \cos(\theta-\beta)\, q(t) \tag{67}$$

where R is the radius of the cylinder and q(t) is the time history of the tip deflections due to the response of the lowest mode. q(t) in this example will only be computed for the damping controlled response near the natural frequency. The quasi-static contributions to response are calculated separately. The mean square stress due to first mode vibration along the x axis in response to unidirectional random waves directed along the same axis is

$$\sigma_{do}^2(\beta) = \sigma_{xo}^2 \left[2A\left(\frac{1.875}{\ell}\right)^2 E\, R \cos\beta\right]^2 \tag{68}$$

For $\beta = 0$ this equation is of the form specified in Equation (50).

$$\sigma_{do}^2 = B^2 \sigma_{xo}^2 \tag{50}$$

$$B^2 = \left[2A\left(\frac{1.875}{\ell}\right)^2 E\, R\right]^2 = 3.52\left(\frac{E\, R}{\ell^2}\right)^2 \tag{69}$$

σ_{xo}^2 is the mean square dynamic deflection of the tip computed using Equation (49). The only precaution in the use of the equation is that modal quantities such as the modal mass M_x must have been computed using the same mode shape that was specified here. In this report the normalization is such that $\psi(z = \ell) = 1.0$.

For a hollow steel cylinder immersed in water and an assumed added mass coefficient of 1.0, the mass per unit length of the cylinder for the purpose of calculating the natural frequency is

$$m = \rho_s 2\pi Rt + \pi R^2 \rho_w \quad \text{(dry interior)} \tag{70}$$

$$m = \rho_s 2\pi Rt + 2\pi R^2 \rho_w \quad \text{(flooded interior)} \tag{71}$$

The modal mass M_x for the lowest mode is:

$$M_x = \int_0^h \psi(z)^2 m \, dz \cong .25 \text{ hm} \tag{72}$$

The natural frequency of the lowest mode from Equation (62) is:

$$\omega_x = \frac{3.52}{h^2} \left(\frac{E}{\rho_w}\right)^{1/2} \left(\frac{R^3 t}{\rho_{s/\rho_w} \; 2\pi Rt + \pi R^2}\right)^{1/2} \tag{73}$$

where ρ_s/ρ_w is the specific gravity of steel.

The preceeding equation is for a dry interior. For a flooded cylinder the πR^2 term in the preceeding equation must be doubled.

If one assumes that the mode shape does not change then an approximate expression for the natural frequency of a cylinder with a tip mass is given by

$$\omega_x' = \omega_x \left(\frac{M_x}{M_o + M_x}\right)^{1/2} \tag{74}$$

where

ω_x = natural frequency without a tip mass

M_x = modal mass from Equation (72) with or without a flooded interior, as appropriate

M_o = tip mass

Therefore, an approximate modal mass including the tip weight is

$$M_x' = M_x + M_o = .25\text{hm} + M_o \tag{75}$$

THE FATIGUE LIFE OF A SINGLE CYLINDER: NUMERICAL EXAMPLES

Two specific numerical examples are evaluated below:

Example 1. Dry interior, no tip mass.

$$h = 70 \text{ m}$$
$$R = 2.1 \text{ m}$$
$$t = .07 \text{ m}$$
$$\rho_s/\rho_w = 7.5$$
$$\rho_w = 1000 \text{ kg/m}^3$$
$$(E/\rho_w)^{1/2} = 1.42 \times 10^4 \text{ m/s}$$
$$\omega_x = 3.19 \text{ rad/s}$$
$$\tau_x = 2\pi/\omega_x = 1.97 \text{ s}$$
$$m = 2.08 \times 10^4 \text{ kg/m}$$
$$M_x = .25hm = 3.64 \times 10^5 \text{ kg}$$
$$B^2 = 3.52 \frac{ER}{\ell^2}^2 = 2.62 \times 10^{16} \left(\frac{nt}{m^2}\right)^2 \Big/ m^2$$
$$\sigma_{do}^2 = B^2 \sigma_{xo}^2 = 2.62 \times 10^{16} \sigma_{xo}^2 \left(\frac{nt}{m^2}\right)^2$$

where σ_{xo}^2 is the mean square deflection from Equation (49) in m^2.

Example 2. Flooded cylinder with a tip mass. Same values as before except as noted.

$$m = 3.46 \times 10^4 \text{ kg/m}$$
$$M_x = .25hm + M_o = 6.06 \times 10^5 + M_o$$
$$M_o = 5.38 \times 10^5$$
$$M_x = 1.14 \times 10^6 \text{ kg}$$
$$\tau_x = 3.5 \text{ s}$$
$$\omega_x = 1.8 \text{ rad/sec}$$

To complete both examples an estimate of σ_{xo}^2 is required from Equation (49). An upper bound on the value of the point wave spectrum at the natural frequency may be easily obtained from Krogstad's formula for the equilibrium limit given in Equation (17).

$$S_\eta(\omega) = 1/2\pi \times 1.62 \times 10^{-3} \left(\frac{\omega}{2\pi}\right)^{-4.6} \tag{17}$$

Evaluating this at the two natural frequencies found above

$$S_\eta(\omega = 3.19) = 5.8 \times 10^{-3}\ m^2\text{-}s$$

$$S_\eta(\omega = 1.8) = 8.1 \times 10^{-2}\ m^2\text{-}s$$

Placing these values in Equation (49) yields
for $\omega = 3.19$

$$\sigma_{xo}^2 = 2.28 \times 10^{-4}\ m^2$$

$$\sigma_{do}^2 = B^2\sigma_{xo}^2 = 5.95 \times 10^{12} \left(\frac{nt}{m^2}\right)^2 = 5.95\left(\frac{nt}{mm^2}\right)^2$$

$$\sigma_{do} = 2.44 \times 10^6 \left(\frac{nt}{m^2}\right) = 2.44\left(\frac{nt}{mm^2}\right)$$

for $\omega = 1.8$

$$\sigma_{xo}^2 = 1.77 \times 10^{-2}\ m^2$$

and $$\sigma_{do}^2 = 4.63 \times 10^{14} \left(\frac{nt}{m^2}\right)^2 = 4.63 \times 10^2 \left(\frac{nt}{mm^2}\right)^2$$

$$\sigma_{do} = 2.15 \times 10^7 \left(\frac{nt}{m^2}\right) = 21.5\left(\frac{nt}{mm^2}\right)$$

As a simple demonstration we may take the worst case example of $R_r(\omega_x)/R_t(\omega_x) = 1.0$ and no directional spreading. Assuming for the purpose of demonstration only that there is no quasi-static contribution to the stress, a fatigue life estimate for the two cases may be achieved using Equation (1).

$$F = \frac{\nu_o^+}{c} \left(2^3 \sigma_s^2\right)^{b/2} \Gamma(1+b/2)$$

where $\nu_o^+ = \dfrac{\omega_x}{2\pi}$

$b = 4.1$

$\log_{10} c = 13.57$

where b and c are taken from DnV guidelines for tubular joints and are based on stress range in nt/mm^2.

$\sigma_s^2 = \sigma_{do}^2$

$\Gamma(1+b/2) = 2.0957$

For the case $\omega_x = 3.19$ rad/s, $\nu_o^+ = .508$ Hz :

$F = 7.87 \times 10^{-11} = .0025/\text{year}$

$1/F = 400$ years fatigue life

For the case $\omega_x = 1.8$ rad/s :

$F = 3.34 \times 10^{-7}/\text{sec} = 10.54/\text{year}$

$1/F = 0.095$ years fatigue life

This result is consistent with Equation (20) which reflects the change in fatigue damage due to a change in natural frequency which results from a change in structural mass, not stiffness.

By including the influence of a realistic level of damping and spreading, a revised estimate of this overly conservative fatigue life estimate may be obtained. From Equation (54), it can be seen that cosine squared spreading reduces the mean square dynamic stress at $\beta = 0$ to 3/4 of σ_{do}^2. From Equations (49) and (50) it can be seen that a value of $R_r/R_t = 1/2$ reduces σ_d^2 by an additional factor of 1/2. This value for damping is found in the following section. By including these effects the revised estimate is

$$\sigma_d^2(\beta = 0) = (3/4)(1/2)\sigma_{do}^2 = 3/8\sigma_{do}^2.$$

Using this value of stress in Equation (1) increases the fatigue life by a factor of 7.47.

For $\omega_x = 3.19$ rad s

$$1/F = 7.47 \times 400 = 2988 \text{ years.}$$

For $\omega_x = 1.8$ rad/sec

$$1/F = 7.47 \times .095 = .71 \text{ years.}$$

THE DAMPING OF A VERTICAL CYLINDER

The mean square dynamic stress is proportional to the ratio of radiation to total damping. This may be expressed in terms of the actual modal damping coefficients or in terms of their equivalent damping ratios as shown.

$$\frac{R_r(\omega_x)}{R_T(\omega_x)} = \frac{\xi_r(\omega_x)}{\xi_T(\omega_x)} \tag{76}$$

The total modal damping is made up of numerous components. They include structural hysteretic, soils, hydrodynamic viscous, and radiation (wave making) damping. Non-linear effects in general will require that the soils and viscous components be adjusted for each sea state. At any given sea state it is assumed that an equivalent linear damping may be found for each component. In the absence of accurate information on the various components the upper bound value of 1.0 may be used for the ratio.

For the vertical cylinder vibrating in the first mode, it is possible to estimate the radiation, viscous, and structural hysteretic components.

The linear radiation damping of the lowest flexural mode of a cylinder may be computed by solving the linear potential flow radiation problem. The solution may be found in a report by Petrauskas [8] and is given below for the case of deepwater waves.

$$R_r = 2\pi R\rho_w \ \omega_x \frac{P_1(ka)}{k^2} \left(1 - \frac{\pi}{kh}\right)^2 \qquad (77)$$

where $a = R$ the radius

$k = \omega_x^2/g$ the wave number

$P_1(ka)$ is a function defined below and plotted in Figure 5.

$$P_1(ka) = 2/\{\pi ka[J_1'(ka)^2 + y_1'(ka)^2]\} \qquad (78)$$

where: J_1 and y_1 are Bessel functions of the first and second kind. The ' indicates the first derivative.

For $ka < 1/2$ $\quad P_1(ka) \simeq \frac{\pi}{2}(ka)^3$

$ka > 2$ $\quad P_1(ka) \simeq 1.0$

Recognizing that

$$\xi_r = \frac{R_r}{2\omega_x M_x} \qquad (79)$$

ξ_r may be calculated for the two specific example problems.

For $\omega_x = 3.19$ rad/s, $\xi_r = .017$

$\omega_x = 1.8$ rad/s, $\xi_r = .018$

The viscous hydrodynamic damping coefficient per unit length has been shown by Dunwoody to be approximately given by

$$R_v = 1/2\rho_w D C_D \sqrt{\frac{8}{\pi}}\,\sigma_{\dot{r}} \tag{80}$$

where D is the cylinder diameter, C_D is the drag coefficient and σ_r is the root mean square relative velocity. Because of the attenuation of waves with depth, $\sigma_{\dot{r}}$ is a function of z. The modal damping coefficient for the first bending mode may be calculated from the following:

$$R_{vx} = \int_0^h 1/2\rho_w C_D D\sqrt{\frac{8}{\pi}}\,\sigma_{\dot{r}}(z)\,\psi(z)^2 dz \tag{81}$$

R_{vx} can only be found by an iterative solution of the equations of motion, because of the relative velocity term. However, an approximate solution may be obtained by assuming that the fluid velocities are much greater than the structural velocities. In other words that $\sigma_u(z) \simeq \sigma_{\dot{r}}(z)$ where $\sigma_u(z)$ is the root mean square water particle velocity. Assuming airy waves and an exponential decay of horizontal water particle velocities with depth an approximate value of $\sigma_u^2(z)$ may be obtained.

$$\sigma_u^2(z) = \int_0^\infty \omega^2 S_\eta(\omega)\; e^{2k(z-h)} d\omega \tag{82}$$

where $k = \omega^2/g$.

Even this integral can be rather difficult to evaluate and therefore to obtain a rough estimate for the examples here, further simplifications were necessary. The first was to assume that the exponential decay of the velocity spectrum is governed by the frequency of waves corresponding to the peak of the wave spectrum, ω_p. The second simplification was to assume that for frequencies above the peak frequency of the wave spectrum, the spectrum may be approximated by the Krogstad equilibrium formulation from Equation (17).

$$S_\eta(\omega) = \frac{1.62\text{x}10^{-3}}{2\pi}\left(\frac{\omega}{2\pi}\right)^{-4.6} \qquad \text{for } \omega > \omega_p \tag{17}$$

$$= 0 \qquad \text{for } \omega < \omega_p$$

With all of the above approximations $\sigma_u^{\ 2}$ may be expressed as:

$$\sigma_u^{\ 2} \simeq \int_{\omega_p}^{\infty} \omega^2 \, \frac{1.62\text{x}10^{-3}}{2\pi}\left(\frac{\omega}{2\pi}\right)^{-4.6} e^{\left(\frac{2\omega_p^{\ 2}}{2}(z-h)\right)} d\omega \tag{83}$$

$$\sigma_u^{\ 2} \simeq 10^{-3} \, e^{\left(\frac{2\omega_p^{\ 2}}{2}(z-h)\right)} \left(\frac{\omega_p}{2\pi}\right)^{-1.6} \tag{84}$$

R_{vx} may be calculated by substituting σ_u into Equation (81) for $\sigma_{\dot{r}}$. This was done for ω_p = .628 rad/s which corresponds to waves with a 10 second period. After dividing by $2\omega_x M_x$ to obtain the damping ratios the approximate results were

for ω_x = 3.19	$\xi_v \simeq .004$	
	$\xi_r = .017$	
ω_x = 1.8	$\xi_v \simeq .002$	
	$\xi_r = .018$	

In both cases the viscous damping was rather small compared to the radiation damping. This is to be expected in a problem for which the inertial forces are large compared to the drag forces. Just as the radiation damping is related to the inertial component of the wave force spectrum, the viscous damping is related to the drag component of the wave force spectrum. In other words, when drag forces cannot be neglected, neither can the viscous element of the damping be neglected. When inertia forces are dominant it is likely that the radiation damping will be much larger than the viscous damping.

The soils damping, ξ_s, is very dependent on site conditions, and no attempt is made to estimate it here. For the sake of example, it will be specified as 1%.

The only remaining element is the structural hysteretic. Based on evaluation of the average strain energy lost per cycle of motion, the hysteretic losses in steel imply 0.25 to 0.5% damping (ξ_h = .0025 to .005).

For the two example structures the damping may be summarized:

For $\omega_x = 3.19$

$$\xi_r = .017$$
$$\xi_v = .004$$
$$\xi_s = .01$$
$$\xi_h = .005$$
$$\frac{\xi_r}{\xi_T} = \frac{.017}{.036} = .47$$

For $\omega_x = 1.8$

$$\xi_r = .018$$
$$\xi_v = .002$$
$$\xi_s = .01$$
$$\xi_h = .005$$
$$\frac{\xi_r}{\xi_T} = \frac{.018}{.035} = .51$$

The total damping in each case is on the order of 3.5% and the ratio of radiation to total damping is approximately 0.5. If the soils damping is actaully only 0.5% instead of 1%, then the total damping is reduced by 14%, but the ratio of radiation to total damping increases by only 8%. The ratio is less sensitive to errors in the individual components than the total damping value.

CONCLUSIONS

By means of general formulations and a specific example, the dependence of fatigue on the uncertainties related to natural frequencies, damping ratios and wave spreading have been demonstrated.

Uncertainties related to the prediction of structural natural frequencies are primarily related to the structural idealizations or models used in the design process. The greatest weakness is probably in the area of foundation modelling. The behavior of soil under cyclic loading conditions remains a rather uncertain field. Assumptions regarding soils stiffness have dramatic impact on the estimation of structural natural frequencies.

The uncertainties related to damping estimates have several sources. One of the greatest is a general lack of accurate estimates of damping on existing structures. This issue and a method for obtaining improved measurements of damping on existing structures are addressed by Campbell [9]. The second reason for uncertainty is that direct estimation of individual components of damping are rarely made, and the knowledge required for making such estimates is not widely available in the industry. To understand the complete damping problem one must understand the fluid mechanics, the soil mechanics, the structural mechanics, and their interaction. A final source of misuse of damping is that the relationships between exciting forces and damping mechanisms are too frequently ignored. In the literature one finds examples where viscous damping is assumed negligibly small, but that drag forces could not be neglected. Such assumptions are inconsistent and reflect a lack of understanding regarding the relationship between hydrodynamic excitation and fluid damping mechanisms.

The significance of wave spreading has been recognized for years. However, the principal weakness in accounting for spreading in dynamic response problems lies with the lack of knowledge of the extent of spreading in realistic sea conditions. Until more accurate descriptions of the sea become available, estimation of spreading for the purposes of dynamic response calculation will remain rather uncertain.

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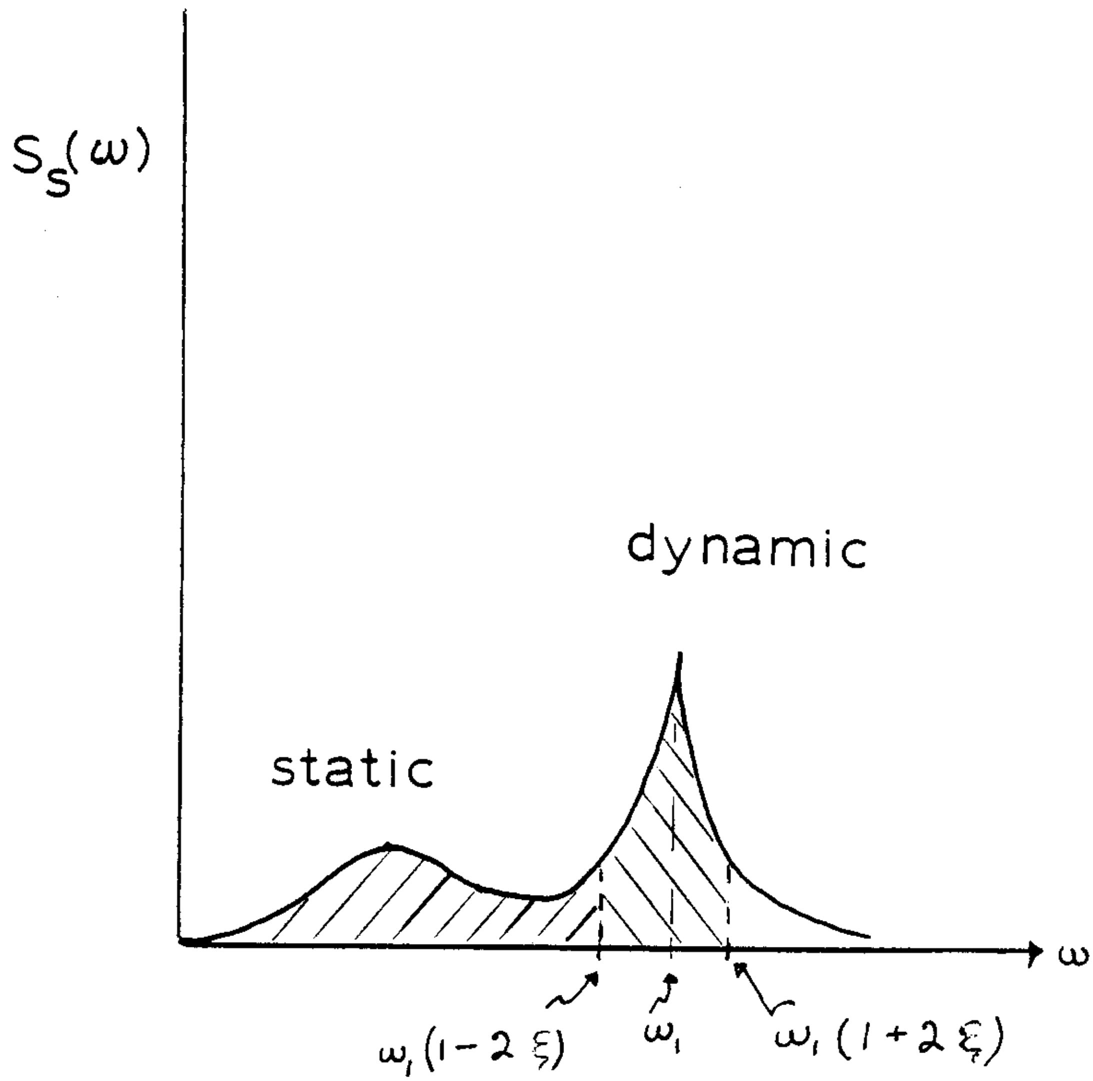

FIGURE 1. The Partitioning of Stress into Static and Dynamic Components.

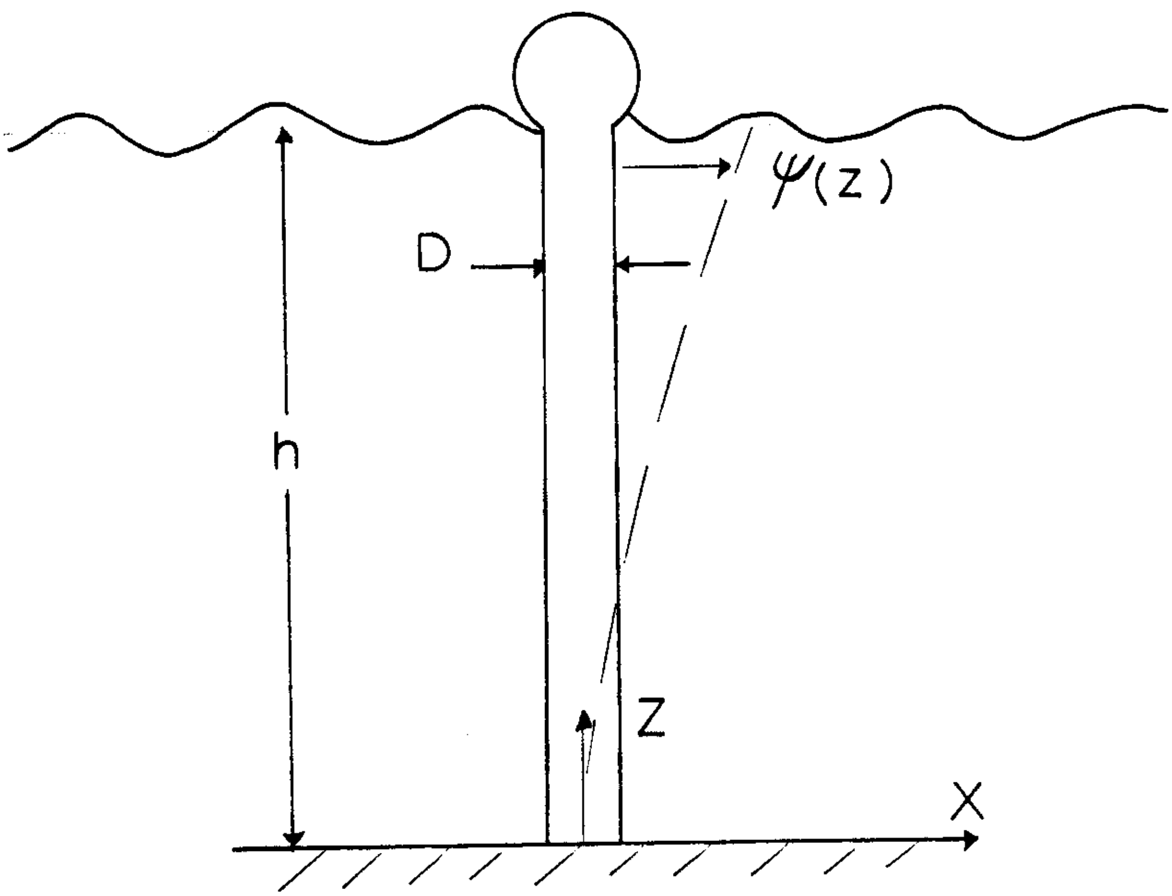

FIGURE 2. Simple Cylindrical Cantiliver

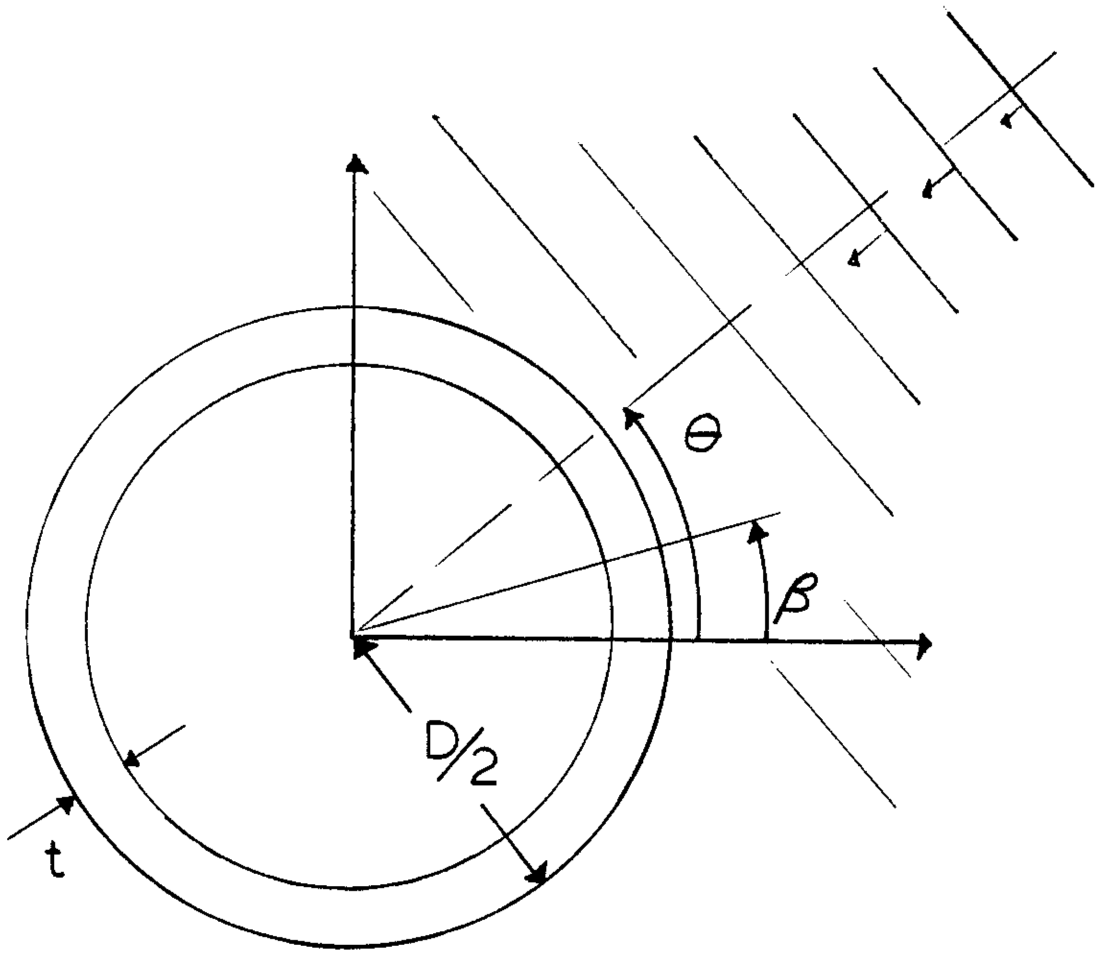

FIGURE 3. Cross Section of Cylinder with Coordinates.

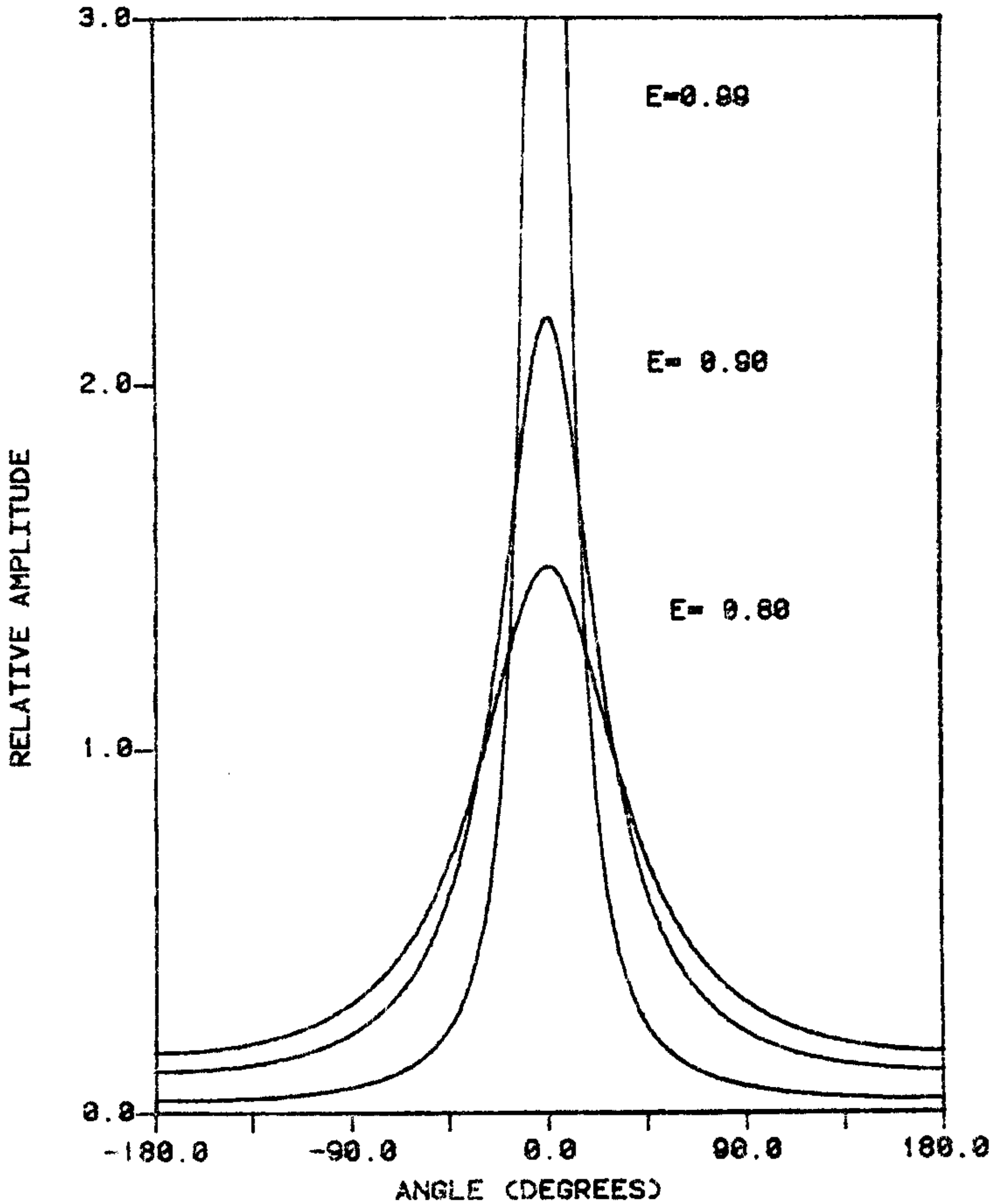

FIGURE 4: ANGULAR SPREADING FUNCTION FOR VARIOUS VALUES OF E

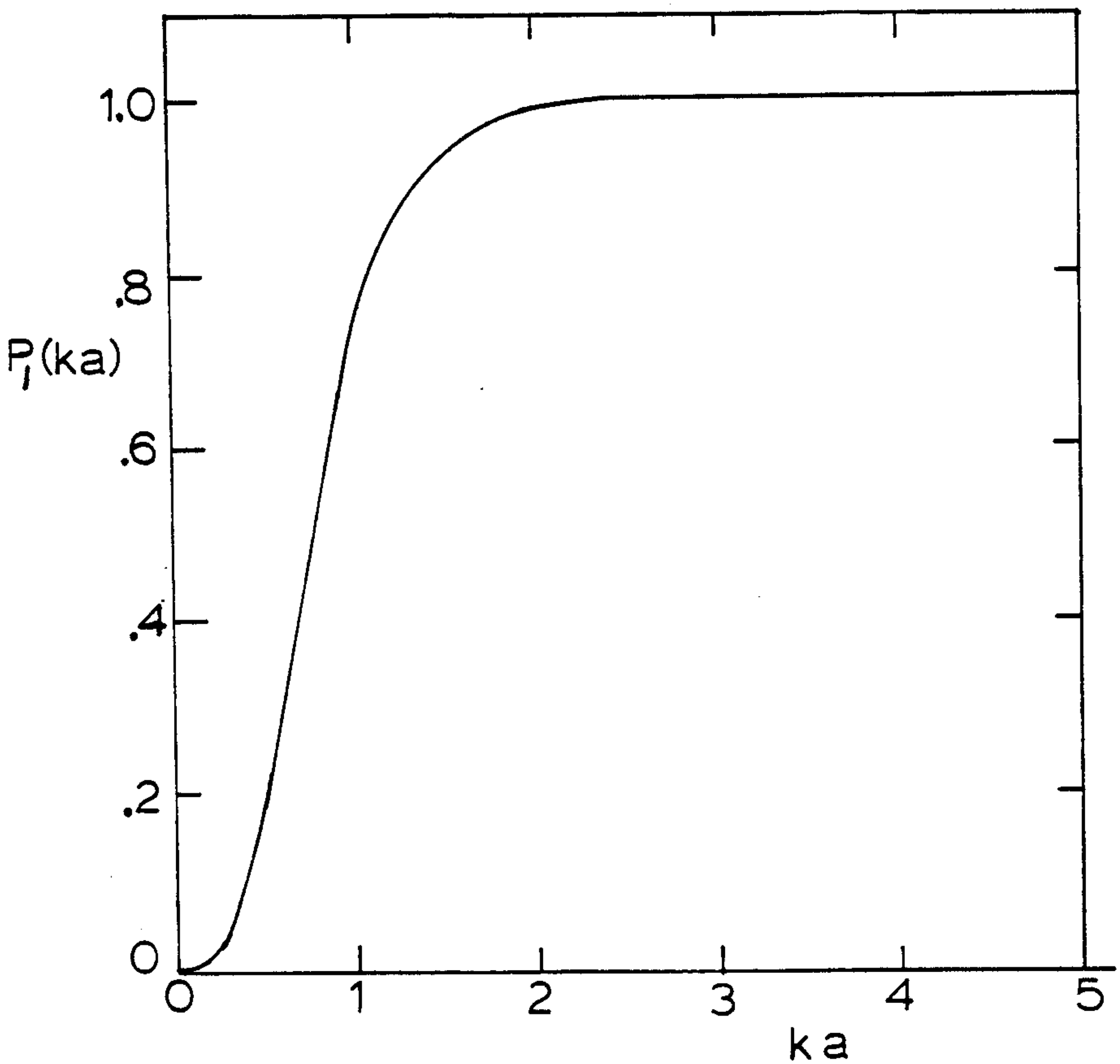

FIGURE 5. $P_1(ka)$